WORKSHEET: SUMMARY OF LAST OF CH 3

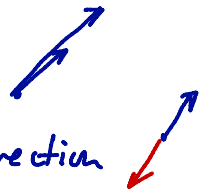
1. For two n-dimensional vectors a and b, we defined the angle, θ, between them to be

$$\theta = \arccos\left(\frac{a^T b}{\|a\|\,\|b\|}\right)$$

2. This is a plausible definition because

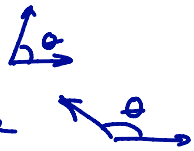
$\left|\frac{a^T b}{\|a\|\,\|b\|}\right| \le 1$ and it's clearly correct for dimensions 2 and 3

3. Fill in the blanks below assuming that a and b are n-dimensional vectors.



(a) $\frac{a^T b}{\|a\|\|b\|} = 1$ if and only if a and b are in the same direction

(b) $\frac{a^T b}{\|a\|\|b\|} = -1$ if and only if a and b are in the opposite direction



(c) $a^T b > 0$ if and only if the angle between them are acute

(d) $a^T b < 0$ if and only if the angle between them is obtuse

(e) $a^T b = 0$ if and only if $a \perp b$

4. Suppose $a = (1, 2, 3, 4)$ and $b = (2, 0, -1, 2)$ and $L(t) = (1 - t)a + tb$ where t is a real number.

(a) Find $L(0)$ and state what *type* of object it is.

$L(0) = (1-0)a + 0b = a$ ← a vector or a point

(b) Find two other L-values.

$L(1) = (1-1)a + 1\cdot b = b$

$L(2) = (1-2)a + 2b = 2b - a = (4,0,-2,4) - (1,2,3,4) = (3,-2,-5,0)$

(c) Rewrite L in the form $L(t) = ct + d$ and explain how you know L is a line.

$L(t) = (1-t)a + tb = a + (b-a)t = (1,2,3,4) + (1,-2,-4,-2)t$

For a given change in t, the change in L is consistent.