WORKSHEET: FACT A, BASIS, AND ORTHONORMAL VECTORS

1. Fact A:

2. Definition: A *basis* is

3. Give three distinct examples of bases when

   (a) $n = 2$

   (b) $n = 3$

4. Fact B:

5. A set of n-vectors $a_1, a_2, \cdots, a_k$ is called *orthogonal* if

6. Examples

7. A vector a is called *normal* if

8. Examples

9. A set of n-vectors $a_1, a_2, \cdots, a_k$ is called *orthonormal* if

10. Examples

11. Suppose a_1, a_2, a_3, and a_4 is a set of orthonormal 32-vectors. Further, suppose that $\beta_1, \beta_2, \beta_3$ and β_4 have the property that

$$\beta_1 a_1 + \beta_2 a_2 + \beta_3 a_3 + \beta_4 a_4 = 0_{32}.$$

(a) Find $a_3^T(\beta_1 a_1 + \beta_2 a_2 + \beta_3 a_3 + \beta_4 a_4)$.

(b) Find $a_3^T 0_{32}$.

(c) What can you conclude about β_3? About β_i for $i = 1, 2, 4$?

(d) What can you conclude about the set a_1, a_2, a_3, and a_4? About *any* set of orthonormal vectors?