WORKSHEET: MATRICES

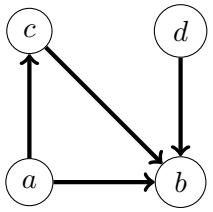
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -5 & 6 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 5 \\ 1 & 2 & -1 \end{bmatrix}, \qquad C = \begin{bmatrix} 3 & 4 \\ 5 & -2 \\ \pi & \sqrt{2} \\ 0 & -7 \end{bmatrix}, \qquad x = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \qquad y = \begin{bmatrix} 3 & 2 & -1 \end{bmatrix}$$

1. How to think about a general $m \times n$ matrix A

2. Applications

   (a) $$\begin{aligned} x_1 + 2x_2 &= 3 \\ 4x_1 - 5x_2 &= 5 \end{aligned}$$

   (b) graph example



3. Special Matrices

(a) $\mathbf{0} = \mathbf{0}_{m \times n}$, the zero matrix

(b) an $n \times n$ square matrix A and its main diagonal versus its off-diagonal

(c) a diagonal matrix D

(d) I_n, the $n \times n$ identity matrix

(e) an upper (lower) triangular matrix A

(f) a block matrix and its submatrices

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4. Things we can do with matrices

   (a) transpose

   (b) matrix addition

   (c) scalar multiplication

   (d) These operations are well-behaved.

(e) **matrix-vector multiplication**