Ch 3

v - vector

the norm of v $= \|v\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2} = v'v$

for $\quad v = (2, -1, 3), \quad w = (-5, 0, 10), \quad \beta = -2$

find

- $\|v\| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{4+1+9} = \sqrt{14} \approx 3.7$
- $\|\beta v\| = \|(-4, 2, -6)\| = \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{16} \approx 7.4$
- $\|w\| = \sqrt{25+0+100} = \sqrt{125} = 5\sqrt{5} \approx 11.8$
- $\|v+w\| = \sqrt{(-3)^2 + (-1)^2 + (13)^2} = \sqrt{179} \approx 13.4$
- $\|v-w\| = \sqrt{8^2 + (-1)^2 + (-7)^2} = \sqrt{114} \approx 10.7$

$13.4 \leq 3.7 + 11.8$ ✓

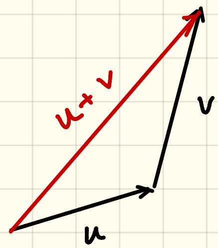
Properties of this norm:

- $\|v\| \geq 0$
- If $\|v\| = 0$, then $v = 0$
- $\|\beta v\| = |\beta| \, \|v\| \qquad \left(\sqrt{\beta^2} = |\beta|\right)$

Not obvious.

- ★ Δ-ineq. $\|v+w\| \leq \|v\| + \|w\|$

geometric intuition for Δ-inequality:



- algebra
- rms(v)
- distance

- Recall innerproduct algebra

show

$$\|\alpha x+y\|^2 = (\alpha x+y)'(\alpha x+y)$$
$$= (\alpha x)'(\alpha x) + (\alpha x)'y + y'(\alpha x) + y'y$$
$$= \alpha^2 x'x + 2\alpha x'y + y'y$$
$$= \alpha^2\|x\|^2 + 2\alpha x'y + \|y\|^2$$

<u>def</u>: Given two n-vectors (points in n-space) x and y

the euclidean distance between x and y

$= \text{dist}(x,y) = \|x-y\| \; (= \|y-x\|)$

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Given vectors $x, \underbrace{a_1, a_2, a_3, \dots, a_k}_{\text{data feature vectors.}}$, k- vv large • Nearest neighbor

new case

Find a_i (or a_i's) s. that $\text{dist}(x, a_i)$ is minimized.

book example

Defn root-mean-square value of v

$$= \mathrm{rms}(v) = \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_n^2}{n}} = \frac{\|v\|}{\sqrt{n}} = \text{(intuitively) averge of } |x_i|$$

vector	$\mathrm{rms}(v)$	$\|v\|$	$\frac{1_n' v}{n}$	$\mathrm{std}(v)$
$(1,1,1,1)$	$\frac{\sqrt{4}}{\sqrt{4}} = 1$	2	1	0
$(-1,1,-1,1)$	1	2	0	1
$(\sqrt{2},\sqrt{2})$	$\frac{\sqrt{4}}{\sqrt{2}} = \sqrt{2}$	2	$\sqrt{2}$	0
$(-\frac{1}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2},1,1,1)$	$\frac{\sqrt{4}}{\sqrt{7}} = \frac{2}{\sqrt{7}} \approx 0.75$	2	$\frac{1}{3}$	$(\frac{-5}{6},\frac{-5}{6},\frac{1}{6},\frac{1}{6},\frac{2}{3},\frac{2}{3},\frac{2}{3})$ 0.62994...

Recall: • $\frac{1_n' v}{n}$ = average value on an entry in v $= \frac{1}{n}\sum_{i=1}^{n} x_i$

$= \mathrm{avg}(v)$ $(\equiv \mu)$ from Stats

• standard deviation ≡ typical distance from mean (intuition)

$$\mathrm{std}(v) = \sqrt{\frac{\sum_{i=1}^{n}(v_i - \mu)^2}{n}} =$$

- standard deviation $\equiv$ typical distance from mean (intuition)

$$\mathrm{std}(v) = \sqrt{\frac{\sum_{i=1}^{n}(v_i - \mu)^2}{n}} = \sqrt{\frac{\sum (v_i - \mathrm{avg}(v))}{n}}$$

$$= \frac{\sqrt{(v_1 - \mathrm{avg}(v))^2 + (v_2 - \mathrm{avg}(v))^2 + \dots + (v_n - \mathrm{avg}(v))^2}}{\sqrt{n}}$$

$$= \frac{\| v - (\mathrm{avg}(v))\, 1_n \|}{\sqrt{n}}$$

$v - \mathrm{avg}(v) 1_n \equiv$ the de-meaned vector