Summary of Null Space and Geometry Worksheet

Ex1 $A = \begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix}$

$f: \mathbb{R}^2 \to \mathbb{R}^2$ by

$f(x) = Ax$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Ex2 $B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

$g: \mathbb{R}^2 \to \mathbb{R}^2$ by $g(x) = Bx$

These maps treat certain vectors by (just) multiplying by a constant.

$$f\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$f\left(\begin{bmatrix} 1 \\ 10 \end{bmatrix}\right) = \begin{bmatrix} 21 \\ 210 \end{bmatrix} = 21 \begin{bmatrix} 1 \\ 10 \end{bmatrix}$$

$$g\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$g\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 3 \end{bmatrix} = 3 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Consequently, these maps treat all vectors in a simple, easy-to-understand ways.

Consider $v = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$.

$$v = \begin{bmatrix} 5 \\ 8 \end{bmatrix} = 2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 10 \end{bmatrix} = 2u + w$$

$$f(v) = Av = A(2u + w)$$

$$= 2Au + Aw$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 21 \\ 210 \end{bmatrix} = \begin{bmatrix} 21 \\ 210 \end{bmatrix}$$

$v = 5e_1 + 8e_2$. So...

$$g(v) = A\,v = A(5e_1 + 8e_2)$$

$$= 5Ae_1 + 8Ae_2$$

$$= 5 \begin{bmatrix} 2 \\ 0 \end{bmatrix} + 8 \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ 24 \end{bmatrix}$$

Is this alway possible?