start here

- Standard deviation ≡ typical distance from mean (intuition)

$$\operatorname{std}(v) = \sqrt{\frac{\sum_{i=1}^{n}(v_i-\mu)^2}{n}} = \sqrt{\frac{\sum\left(v_i - \operatorname{avg}(v)\right)^2}{n}}$$

$$= \frac{\sqrt{(v_1-\operatorname{avg}(v))^2 + (v_2-\operatorname{avg}(v))^2 + \dots + (v_n - \operatorname{avg}(v))^2}}{\sqrt{n}}$$

$$= \frac{\| v - (\operatorname{avg}(v))\, 1_n \|}{\sqrt{n}}$$

$v - \operatorname{arg}(v) 1_n$ ≡ the de-meaned vector

skip...

Algebra: $(\operatorname{rms}(x))^2 = (\operatorname{avg}(x))^2 + (\operatorname{std}(x))^2$

The rest of Ch3

- Cauchy-Schwartz Inequality
- angle between vectors
- Lines in n-dim'l space
- Chebyshev (?)

Cauchy-Schwartz Inequality

a, b vectors $\quad |a^T b| \leq \|a\| \|b\|$

ex: $\overset{a}{(1,2,-3)}, \overset{b}{(-2,0,1)}$

$$|a^T b| = |-2+0-3| = 6$$

$$\|a\| = \sqrt{1+4+9} = \sqrt{14}, \quad \|b\| = \sqrt{4+1} = \sqrt{5}$$

$$\|a\| \|b\| = \sqrt{70} > 6$$

logic

Recall Precalc ☺ $|x| \le L$ means

$-L \le x \le L$ or

$L \ge -x$ and $x \le L$

$\rightarrow$ x and $-x$ are bounded above by L

Why? (Proof) Let $\alpha = \|a\|$, $\beta = \|b\|$

$$0 \le \|\beta a - \alpha b\|^2 = (\beta a - \alpha b)^T(\beta a - \alpha b)$$

foil

$$= \beta^2\|a\|^2 - 2\alpha\beta a^T b + \alpha^2\|b\|^2$$

$\alpha = \|a\|$
$\beta = \|b\|$

$$= \|b\|^2\|a\|^2 - 2\|a\|\|b\| a^T b + \|a\|^2\|b\|^2$$

$$= \cancel{2}\|a\|^2\|b\|^2 - \cancel{2}\|a\|\|b\| a^T b$$

collect terms. $\div$ by 2

re arrange

So $\|a\|\|b\| a^T b \le \|a\|^2\|b\|^2$

So $a^T b \le \|a\|\|b\|$

To get the 2^{nd}, start $0 \le \|-(\beta a - \alpha\beta)\|^2$

teeny aside: Use it in #20

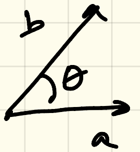
Why do we care?

If $|a^T b| \le \|a\|\|b\|$, then

$$\boxed{-1 \le} \; \frac{a^T \cdot b}{\|a\|\|b\|} \; \boxed{\le 1}$$

That is: $\operatorname{acos}\left(\frac{a^T b}{\|a\|\|b\|}\right)$ makes sense

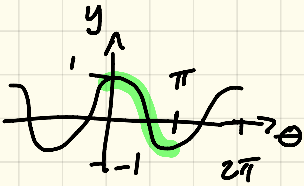
def: a, b vectors, θ angle between a and b



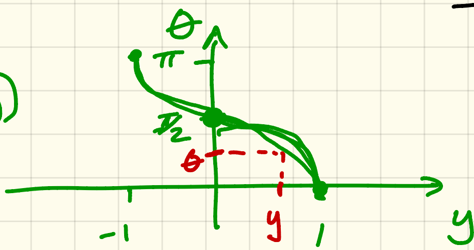
$$\theta = \operatorname{acos}\left(\frac{a^T b}{\|a\|\,\|b\|}\right)$$

better be between -1 and 1 !!

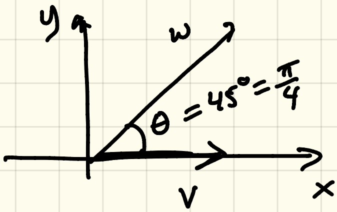
Go back to Trig: $f(\theta) = \cos(\theta)$



$f^{-1}(y) = \operatorname{acos}(y)$



Ex



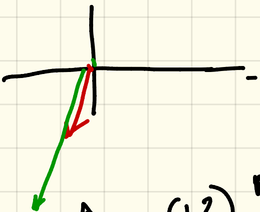
$v = (5, 0)$

$w = (3, 3)$

$$\frac{v^T w}{\|v\|\|w\|} = \frac{15 + 0}{5 \cdot \sqrt{18}} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

So $\theta = \arccos\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

Ex



$v = (-1, -2)$

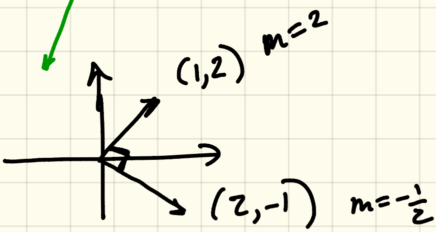
$w = (-2, -4)$

$$\frac{v^T w}{\|v\|\|w\|} = \frac{2 + 8}{\sqrt{5} \cdot \sqrt{20}}$$

$$= \frac{10}{10} = 1$$

$\theta = \arccos(1) = 0$

Ex



$$\frac{v^T w}{\|v\|\|w\|} = \frac{0}{2\sqrt{5}} = 0$$

$\operatorname{acos}(0) = \frac{\pi}{2}$