Your Name

Your Signature

Problem	Total Points	Score
1	20	
2	12	
3	10	
4	10	
5	15	
6	15	
7	10	
8	8	
extra credit	5	
Total	100	

- You have 2 hours.
- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)
- You may not use a calculator, book, notes or aids of any kind.
- In order to earn partial credit, you must show your work.

1. (20 points)

   (a) State the negation of each statement below.

      i. If n is divisible by 14, then n is divisible by 2 and n is divisible by 7.

      ii. For every real number r there exists a rational number q such that $r < q < r+1$.

   (b) Determine the truth value of the statements below.

      i. $2 \in \mathcal{P}(\{0, 1, 2, 3\})$

      ii. $\{\emptyset, \{0, 1\}\} \subseteq \mathcal{P}(\{0, 1, 2, 3\})$

   (c) List three different partitions of the set $S = \{1, 2, 3\}$. Label your partitions P_1, P_2, and P_3. Use correct notation.

   (d) Let R be an equivalence relation on $S = \{a, b, c, d\}$ such that aRb and dRa. Circle all of the following statements that *must* also be true.

      i. cRc

      ii. bRd

      iii. $d \in [a]$

2. (12 points) Prove that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ for all $n \in \mathbb{N}$.

3. (10 points) Use the method of proof by contrapositive to prove the proposition below.

   Suppose $a, b \in \mathbb{Z}$. If $(a+1)b^2$ is even, then a is odd or b is even.

4. (10 points) Use the method of proof by contradiction to proof the proposition below.

   Suppose $a, b \in \mathbb{R}$. If a is rational and ab is irrational, then b is irrational.

5. (15 points) Let the function $f : [0, \infty) \to [6, \infty)$ be defined as $f(x) = 3x^2 + 6$. Prove that f is a bijection.

6. (15 points) Let R be a relation on $\mathbb{R}$ such that xRy if $x - y \in \mathbb{Z}$.

   (a) Prove that R is an equivalence relation.

   (b) State three distinct elements in $[\pi]$, the equivalence class of R containing π.

7. (10 points) Let A, B, and C be sets. Suppose that $A \subseteq B$, $B \subseteq C$, and $C \subseteq A$. Prove that $A = B$.

8. (8 points) Demonstrate that the sets $\{0, 1\} \times \mathbb{N}$ and $\mathbb{Z}$ have the same cardinality.

5 points extra credit Prove that your answer above is correct.