Your Name

Problem	Total Points	Score
1	8	
2	10	
3	10	
4	10	
5	12	
6	12	
7	15	
8	8	
9	15	
Total	100	

- You have 1 hour.
- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)
- You may not use a calculator, book, notes or aids other than a single 3 by 5 notecard.
- In order to earn partial credit, you must show your work.

1. (8 pts) Write the set using **set builder** notation.
   (a) $\{\ldots, -10, -6, -2, 2, 6, 10, 14, 18, \ldots\}$ (Assume the pattern continues.)

   (b) The set of real numbers whose squares are integers.

2. (10 pts) Consider the set $A = \big\{1,\ 2,\ 3,\ \{2\},\ \{2,3\},\ \{1,\{1\}\}\ \big\}$
   (a) The cardinality of A is _____

   (b) Determine if the statements below are true or false.

   i. $\emptyset \in A$

   ii. $\emptyset \subseteq A$

   iii. $1 \in A$

   iv. $1 \subseteq A$

   v. $\{1\} \in A$

   vi. $\{1\} \subseteq A$

   vii. $\{1,\{1\}\} \subseteq A$

   viii. $\{2,\{2\}\} \subseteq A$

3. (10 pts) Consider the set $B = \{n^2 \,:\, n \in \mathbb{Z} \text{ and } |n| \leq 3\}$
   (a) Rewrite the set B by listing its elements between braces.

   (b) $\big|\mathcal{P}(B)\big| =$ _____ (where $\mathcal{P}(B)$ is the power set of B.)
   (c) Show that the following statement is true:

$$\exists X_1 \in \mathcal{P}(B), \exists X_2 \in \mathcal{P}(B),\ 0 < |X_1| < |X_2| \text{ and } X_1 \cap X_2 = \emptyset.$$

   (d) $\big|B \times B\big| =$ _____
   (e) Let $D = \{(a,b) \in B \times B \,:\, a + b = 1\}$. Rewrite the set D by listing its elements between braces.

4. (10 pts) Let $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : y \leq 1\}$, $B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : y \geq x^2\}$, and the universal set, $U = \mathbb{R} \times \mathbb{R}$, the entire xy-plane. Sketch each set below. Clearly label your pictures.

   (a) $A \cap B$

   (b) $\overline{A} \cup B$

5. (12 pts) For any number n, let $A_n = [0, \frac{n}{n+1}]$. Determine the following subsets of the real line.

   (a) $A_1 =$ _____

   (b) $A_2 =$ _____

   (c) $A_3 =$ _____

   (d) $\bigcap_{n \in \mathbb{N}} A_n =$ _____

   (e) $\bigcup_{n \in \mathbb{N}} A_n =$ _____

6. (12 pts) Rewrite each sentence below in the form "If P, then Q."

   (a) In order for Rachel to wear black, it is necessary that it is a Tuesday.

   (b) The presence of a full moon is sufficient for the flower to bloom.

   (c) The card is an ace only if the table is flat.

   (d) The blueberries are ripe or the cranberries are ripe.

7. (15 pts) For parts (a) and (b): (i) rewrite the given statement symbolically and, then, (ii) negate it.

   (a) For every subset X of $\mathbb{N}$ there is always another subset Y of $\mathbb{N}$ such that $X \neq Y$ and $X \subseteq Y$.

   i. symbolic form:

   ii. negation:

   (b) For every $\epsilon > 0$, there is a $\delta > 0$ such that if $|x - a| < \delta$ and $x \neq a$, then $|f(x) - L| < \epsilon$. (Note: The function $f(x)$ is fixed and a and L represent fixed constants.)

   i. symbolic form:

   ii. negation:

   (c) Determine if the statement in part (a) is true or false. Justify your answer.

8. (8 pts) Show that the statements $\boxed{(P \Rightarrow Q) \Rightarrow R}$ and $\boxed{(P \wedge Q) \vee R}$ are **not** logically equivalent.

9. (15 pts) **Use a truth table** to determine whether the argument below is valid or invalid. **Your answer must include:**

   (a) A complete truth table with clearly labeled columns.

   (b) An explanation of how the truth table demonstrates whether the argument is valid or invalid

   Note: Organize your work clearly. Points will be deducted for poor organization.

$$\begin{array}{l} P \Rightarrow R \\ Q \Rightarrow R \\ \hline \therefore\ P \vee Q \Rightarrow R \end{array}$$