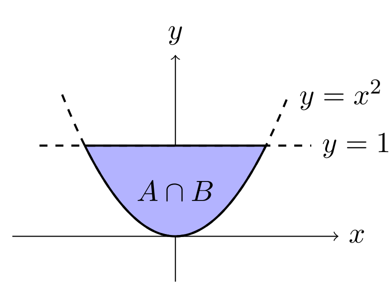
1. (8 pts) Write the set using **set builder** notation.
   (a) $\{\dots, -10, -6, -2, 2, 6, 10, 14, 18, \dots\}$ (Assume the pattern continues.)
   $\{4n + 2 : n \in \mathbb{Z}\}$
   (b) The set of real numbers whose squares are integers.
   $\{x \in \mathbb{R} : x^2 \in \mathbb{Z}\}$

2. (10 pts) Consider the set $A = \big\{1,\ 2,\ 3,\ \{2\},\ \{2,3\},\ \{1,\{1\}\}\big\}$
   (a) The cardinality of A is 6.
   (b) Determine if the statements below are true or false.

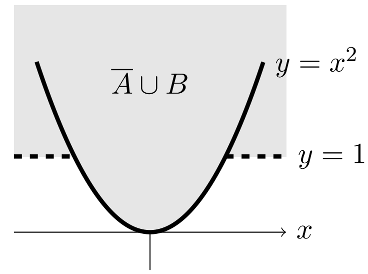
   i. $\emptyset \in A$ False
   ii. $\emptyset \subseteq A$ True
   iii. $1 \in A$ True
   iv. $1 \subseteq A$ False
   v. $\{1\} \in A$ False
   vi. $\{1\} \subseteq A$ True
   vii. $\{1, \{1\}\} \subseteq A$ False
   viii. $\{2, \{2\}\} \subseteq A$ True

3. (10 pts) Consider the set $B = \{n^2 \,:\, n \in \mathbb{Z}$ and $|n| \leq 3\}$
   (a) Rewrite the set B by listing its elements between braces.
   $\{0, 1, 4, 9\}$
   (b) $\big|\mathcal{P}(B)\big| = \underline{2^4 = 16}$ (where $\mathcal{P}(B)$ is the power set of B.)
   (c) Show that the following statement is true:
   $$\exists X_1 \in \mathcal{P}(B), \exists X_2 \in \mathcal{P}(B),\ 0 < |X_1| < |X_2| \text{ and } X_1 \cap X_2 = \emptyset.$$
   Choose $X_1 = \{0\}$ and $X_2 = \{1, 4\}$.
   (d) $\big|B \times B\big| = \underline{4 \cdot 4 = 16}$
   (e) Let $D = \{(a, b) \in B \times B \,:\, a + b = 1\}$. Rewrite the set D by listing its elements between braces.
   $\{(1, 0), (0, 1)\}$

4. (10 pts) Let $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} \,:\, y \leq 1\}$, $B = \{(x, y) \in \mathbb{R} \times \mathbb{R} \,:\, y \geq x^2\}$, and the universal set, $U = \mathbb{R} \times \mathbb{R}$, the entire xy-plane. Sketch each set below. Clearly label your pictures.
   (a) $A \cap B$



(b) $\overline{A} \cup B$



(b) $\overline{A} \cup B$

5. (12 pts) For any number n, let $A_n = [0, \frac{n}{n+1}]$. Determine the following subsets of the real line.

   (a) $A_1 = \underline{[0, \frac{1}{2}]}$

   (b) $A_2 = \underline{[0, \frac{2}{3}]}$

   (c) $A_3 = \underline{[0, \frac{3}{4}]}$

   (d) $\bigcap_{n \in \mathbb{N}} A_n = \underline{[0, \frac{1}{2}]}$

   (e) $\bigcup_{n \in \mathbb{N}} A_n = \underline{[0, 1)}$

6. (12 pts) Rewrite each sentence below in the form "If P, then Q."

   (a) In order for Rachel to wear black, it is necessary that it is a Tuesday.

   If Rachel wears black, then it is Tuesday.

   (b) The presence of a full moon is sufficient for the flower to bloom.

   If there is a full moon, then the flower will bloom.

   (c) The card is an ace only if the table is flat.

   If the card is an ace, then the table is flat.

   (d) The blueberries are ripe or the cranberries are ripe.

   If the blueberries are not ripe, then the cranberries are ripe.

7. (15 pts) For parts (a) and (b): (i) rewrite the given statement symbolically and, then, (ii) negate it.

   (a) For every subset X of $\mathbb{N}$ there is always another subset Y of $\mathbb{N}$ such that $X \neq Y$ and $X \subseteq Y$.

      i. symbolic form: $\forall X \subseteq \mathbb{N}, \exists Y \subseteq \mathbb{N}, (X \neq Y) \wedge (X \subseteq Y)$

      ii. negation: $\exists X \subseteq \mathbb{N}, \forall Y \subseteq \mathbb{N}, (X = Y) \vee (X \nsubseteq Y)$

(b) For every $\epsilon > 0$, there is a $\delta > 0$ such that if $|x - a| < \delta$ and $x \neq a$, then $|f(x) - L| < \epsilon$. (Note: The function $f(x)$ is fixed and a and L represent fixed constants.)

i. symbolic form: $\forall \epsilon > 0, \exists \delta > 0, \forall x \in \mathbb{R}, \big((|x-a| < \delta) \wedge (x \neq a)\big) \Rightarrow (|f(x) - L| < \epsilon)$

ii. negation: $\exists \epsilon > 0, \forall \delta > 0, \exists x \in \mathbb{R}, \big((|x-a| < \delta) \wedge (x \neq a) \wedge (|f(x) - L| < \epsilon)\big)$

(c) Determine if the statement in part (a) is true or false. Justify your answer.

It is false. Or, equivalently, its negation is true. Specifically, if we select $X = \mathbb{N}$, then for every subset Y of $\mathbb{N}$, $Y = X$ or not. If $Y = X$, then the negation is true. If $Y \neq X$, then X necessarily contains an element that Y does not. Thus, $X \not\subseteq Y$.

8. (8 pts) Show that the statements $\boxed{(P \Rightarrow Q) \Rightarrow R}$ and $\boxed{(P \wedge Q) \vee R}$ are **not** logically equivalent.

It is sufficient to find truth values for $P, Q,$ and R such that the statements have different values.

Choose $P =$ true$, Q = R$false.

Then the second one, $\boxed{(P \wedge Q) \vee R}$, is false since R is false by choice and $P \wedge Q$ is false since $T \wedge F = F$.

On the other hand, $(P \Rightarrow Q) = F$ since the hypothesis is true and the conclusion is false. Thus, the first statement now has the form $F \Rightarrow T = T$.

9. (15 pts) **Use a truth table** to determine whether the argument below is valid or invalid. **Your answer must include:**

(a) A complete truth table with clearly labeled columns.

(b) An explanation of how the truth table demonstrates whether the argument is valid or invalid

Note: Organize your work clearly. Points will be deducted for poor organization.

$$\begin{array}{l} P \Rightarrow R \\ Q \Rightarrow R \\ \hline \therefore P \vee Q \Rightarrow R \end{array}$$

	1	2	3	4	5	6	7
row	P	Q	R	$P \Rightarrow R$	$Q \Rightarrow R$	$(P \vee Q)$	$(P \vee Q) \Rightarrow R$
a	T	T	T	T	T	T	T
b	T	T	F	F	F	T	F
c	T	F	T	T	T	T	T
d	T	F	F	F	T	T	F
e	F	T	T	T	T	T	T
f	F	T	F	T	F	T	F
g	F	F	T	T	T	F	T
h	F	F	F	T	T	F	T

There are five rows (a,c,e,g,h) where both hypotheses (columns 4 and 5) are true. For each of these five rows, column 7 is also true. This demonstrates that the argument is valid.