

Your Name

Problem	Total Points	Score
1	15	
2	15	
3	15	
4	20	
5	15	
6	20	
Total	100	

- You have 1 hour.
- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)
- You may not use a calculator, book, notes or aids other than a single 3 by 5 notecard.
- In order to lower the writing load, you may use the symbols \in , \exists , and \forall *if* they are used precisely and literally. You will not be graded on aesthetics, so repeated use of "Thus, ..." will not be penalized.
- You may *not* use the symbol \Rightarrow except parenthetically to indicate the logical structure of an *if-and-only-if* argument.
- You must write in complete sentences with proper punctuation. Every sentence must begin with a capitalized word in English, not a symbol.
- If you are going to use a method *other than a direct proof*, you must state this at the beginning of your proof unless the directions require you to use particular approach.

1. (15 pts) Give a **direct** proof of the statement below.

For every real number x , if $x \geq -1$, then there exists some real number y such that $y^2 - 1 = x$.

2. (15 points) Give a proof by **contrapositive** of the statement below.

For every pair of integers a and b , if $(a + 1)(6a + b)$ is odd, then a is even and b is odd.

3. (15 points) Give a proof by **contradiction** of the statement below.

Let x and y be real numbers. If $5x + 20y = 754$, then either x or y is not an integer.

4. (20 points) Let A and B be sets. Prove that $A \subseteq B$ if and only if $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

5. (15 points) Let A , B , and C be sets. If $A \subseteq B$, then $C - B \subseteq C - A$.

6. (20 points) Let a, b , and c be positive integers. Prove that if $c \mid a$ and $c \mid b$, then $c \mid \gcd(a, b)$.