Your Name

Boiler Plate Language That Everyone Should have written.

Problem	Total Points	Score
1	15	
2	15	
3	15	
4	20	
5	15	
6	20	
Total	100	

- You have 1 hour.
- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)
- You may not use a calculator, book, notes or aids other than a single 3 by 5 notecard.
- In order to lower the writing load, you may use the symbols $\in$, $\exists$, and $\forall$ *if* they are used precisely and literally. You will not be graded on aesthetics, so repeated use of "Thus, ..." will not be penalized.
- ~~You may *not* use the symbol $\Rightarrow$ except parenthetically to indicate the logical structure~~ of an *if-and-only-if* argument.
- You must write in complete sentences with proper punctuation. Every sentence must begin with a capitalized word in English, not a symbol.
- If you are going to use a method *other than a direct proof*, you must state this at the beginning of your proof unless the directions require you to use particular approach.

1. (15 pts) Give a **direct** proof of the statement below.

   For every real number x, if $x \geq -1$, then there exists some real number y such that $y^2 - 1 = x$.

Proof: Suppose $x \in \mathbb{R}$ such that $x \geq -1$.

Now, pick $y = \boxed{}$

Plug $y = \boxed{}$ to y^2-1 to get . . .

2. (15 points) Give a proof by **contrapositive** of the statement below.

For every pair of integers a and b, if $(a+1)(6a+b)$ is odd, then a is even and b is odd.

Pf: Suppose $a, b \in \mathbb{Z}$. We will show that if a is odd OR b is even, then $(a+1)(6a+b)$ is even.

Case 1 : a is odd

Case 2 : b is even

Thus, in both cases, $(a+1)(6a+b)$ is even.

3. (15 points) Give a proof by **contradiction** of the statement below.

   Let x and y be real numbers. If $5x + 20y = 754$, then either x or y is not an integer.

Pf: Suppose $x, y \in \mathbb{R}$ such that $5x+20y = 754$.
Further suppose that both x and y are integers.

Thus, we have the contradiction that ...

4. (20 points) Let A and B be sets. Prove that $A \subseteq B$ if and only if $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

Pf: Suppose A and B are sets

$\Rightarrow$ Suppose $A \subseteq B$. Let $X \in \mathcal{P}(A)$ be arbitrary.

$\vdots$

Thus, $X \subseteq \mathcal{P}(B)$. Thus, $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

$\Leftarrow$: Suppose $\mathcal{P}(A) \subseteq \mathcal{P}(B)$. Let $a \in A$.

$\vdots$

Thus, $a \in B$. Thus, $A \subseteq B$.

5. (15 points) Let A, B, and C be sets. If $A \subseteq B$, then $C - B \subseteq C - A$.

Pf: Let A, B, and C be sets such that $A \subseteq B$.
Let x be an arbitrary element of $C-B$.

Thus, $x \in C-A$. Thus, $C-B \subseteq C-A$.

6. (20 points) Let a, b, and c be positive integers. Prove that if $c \mid a$ and $c \mid b$, then $c \mid \gcd(a, b)$.

Pf: Let $a,b,c \in \mathbb{Z}^+$ such that $c \mid a$ and $c \mid b$.

$\vdots$

Thus, $cl = \gcd(a,b)$.

Thus, $c \mid \gcd(a,b)$.