Your Name

Problem	Total Points	Score
1	20	
2	15	
3	15	
4	32	
5	18	
Total	100	

- You have 1 hour.
- If you have a cell phone with you, it should be turned off and put away. (Not in your pocket)
- You may not use a calculator, book, notes or aids other than a single 3 by 5 notecard.
- In order to lower the writing load, you may use the symbols $\in$, $\exists$, and $\forall$ *if* they are used precisely and literally. You will not be graded on aesthetics, so repeated use of "Thus, ..." will not be penalized.
- You may *not* use the symbol $\Rightarrow$ except parenthetically to indicate the logical structure of an *if-and-only-if* argument.
- You must write in complete sentences with proper punctuation. Every sentence must begin with a capitalized word in English, not a symbol.
- If you are going to use a method *other than a direct proof*, you must state this at the beginning of your proof unless the directions require you to use particular approach.
- When using proof by induction, you *must* indicate when the inductive hypothesis is used.

1. (20 pts)

   (a) (5 pts) Describe the standard method to prove $X \subseteq Y$ for sets X and Y.

   (b) (15 pts) Use the standard method for showing set containment to prove that for all sets A, B, and C, if

   $$A \cap C \subseteq B \cap C \text{ and } A \cup C \subseteq B \cup C,$$

   then $A \subseteq B$.

2. (15 pts) **Use mathematical induction** to prove

$$3^1 + 3^2 + 3^3 + \cdots + 3^n = \frac{3^{n+1} - 3}{2}$$

for all integers $n \in \mathbb{N}$.

3. (15 pts) **Use mathematical induction** to prove $2^n < (n+1)!$ for all integers $n \geq 2$.

4. (8 pts. each for 32 pts. total) Disprove the statements below.

(a) If $X \subseteq \mathbb{N}$ and $|X|$ is infinite, then $|\mathbb{N} - X|$ is finite.

(b) For all sets A, B, and C, if $A \subseteq B$, then $A \cap \overline{(B \cap C)} = \emptyset$.

(c) For every $n \in \mathbb{N}$, there exists some $m \in \mathbb{N}$ such that $mn \equiv 1 \pmod{10}$.

(d) The relation R on $\mathbb{Z}$ defined by xRy if $|x - y| < 5$ is transitive.

5. (18 pts.) Let $A = \{1, 2, 3\}$. Let R be a relation on $\mathcal{P}(A)$, the power set of A, defined as $X\,R\,Y$ if $X \cap \{1, 2\} = Y \cap \{1, 2\}$.

   (a) (2 pts) Show that for $X = \{1\}$ and $Y = \{1, 3\}$, by the definition, $X\,R\,Y$.

   (b) (10 pts) Prove that R is an equivalence relation.

   (c) (6 pts) Describe the partition of $\mathcal{P}(A)$ resulting from the equivalence classes of R. Use proper notation and clearly indicate the members of each partition.