

Preliminaries

The Final Exam will be given without the use of aids of any kind. You will have two hours to complete the test. The exam will cover Chapters 1-2, 4-12, 14. The majority of the problems will be proofs. In some cases you will be asked to use a particular technique to prove a statement. In other instances it will be your choice what method to use. For all proofs you are expected to write complete, formal, appropriately detailed proofs.

Generally, a student will be awarded half the points for a proof problem for writing complete, correct boiler-plate language.

You may bring one 3 by 5 notecard with writing on both sides.

Step 1

Know the formal definitions for the terms below.

1. element of a set, **cardinality of a set** including for infinite sets, set builder notation, natural numbers, integers, rational numbers, irrational numbers, real numbers, interval notation
2. ordered pair, **Cartesian product**, **ordered n -tuple**
3. **subset**, **the power set of a set**
4. **union**, **intersection and difference of two sets**, **complement of a set**
5. a statement, the mathematical meaning of *and*, *or* and *not*, truth table, conditional statement, biconditional, quantifiers, logically equivalent, contrapositive, negation
6. **odd**, **even**, **same parity**, **opposite parity**
7. **divides**, **multiple**, **divisor**
8. **prime**
9. **greatest common divisor**; **least common multiple**
10. **congruent modulo n**
11. **rational number**, **irrational number**
12. **subsets**, **set equality**
13. **relation on A** , **reflexive**, **symmetric**, **transitive relations**, **a relation from A to B** , **an equivalence relation**, **an equivalence class**, **a partition of a set**
14. **a function from A to B** , **domain**, **codomain**, **range of f** , **injective**, **surjective**, **and bijective function**, **Pigeonhole Principle**, **identity function**, **inverse relation**, **inverse function**, **image**, **preimage**

Step 2

Review the boiler-plate wording and standard strategies for proofs of the types below. Double-check in your text if needed.

1. direct proof
2. using cases
3. by contrapositive (As practice you can go to page 155-6 and re-write each conditional statement in its contrapositive form.)
4. by contradiction (As practice you can go to page 155-6 and write what would be the very first line on a proof by contradiction.)
5. if-and-only-if proofs
6. existence statements
7. proofs involving sets (that is, statements including $A \subseteq B$ or $A = B$)
8. How to disprove different kinds of statements.
9. mathematical induction (strong or weak)
10. how to show a relation is or isn't symmetric, reflexive, transitive, an equivalence relation, or a function.
11. how to identify distinct equivalence classes
12. how to prove a function is or isn't injective, surjective, or bijective.
13. how to find an inverse relation
14. how to find the image or preimage of a set given a function
15. techniques to determine if two infinite sets do or do not have the same cardinality
16. techniques to determine if a set is or is not countable

Things to Keep in Mind

1. If a proof technique is not prescribed, you are probably going to be more successful if you state the method you are using.
2. You should put in the “boiler-plate” language even if you cannot figure out the whole proof.
3. You should expect to *use* all of the hypotheses.
4. I will *not* ask you to prove something that is false.
5. One of the cardinal sins in mathematics is using *faulty logic*. Be vigilant against employing any of the approaches below.
 - (a) Assuming what it is you are supposed to be proving.
 - (b) Using the underlying argument: $p \rightarrow q$ is true and q is true, so p must be true.
 - (c) The statement holds for this particular example, thus we can conclude the statement holds in general.
 - (d) Using magic/intimidation to draw an otherwise unwarranted conclusion. Such sentences often begin with “Clearly...”
6. Be skeptical of yourself! Ask yourself if you really *believe* what you wrote down. This is a way of avoiding writing down an assertion that is obviously false.
7. Two answers with exactly the same amount of correct work will not earn the same grade if one of them also contains incorrect work.

What the Exam Will Look Like

- A statement you must prove by contrapositive.
- A statement you must prove by contradiction.
- A statement you must prove by induction.
- A statement you must prove involving sets such as set containment or set equality.
- A question about equivalence relations and equivalence classes
- A question about functions being injective (or not) and surjective (or not)
- A grab-bag short answer question