Your Name Here

February 3, 2026

Math 265

Homework #500

due 1/11/2026

§4.1, #3: Evaluate $\int_1^2 \frac{1}{x^2}\, dx$.

Answer:

$$\int_1^2 \frac{1}{x^2}\, dx = \int_1^2 x^{-2}\, dx = -x^{-1}\Big|_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$$

§4.2, #17 Prove $\sqrt{2}$ is irrational.

Proof. Suppose, to the contrary, that $\sqrt{2}$ is rational. Then

$$\sqrt{2} = \frac{a}{b}$$

where $a, b \in \mathbb{Z}$, $b \neq 0$ with a, b having no common factors. Squaring yields

$$2 = \frac{a^2}{b^2},$$

so

$$2b^2 = a^2.$$

This shows 2 divides a^2, and so since 2 is prime by a lemma proved in class, we see 2 divides a. Letting $a = 2c$ for some $c \in \mathbb{Z}$, this implies

$$2b^2 = 4c^2,$$

so

$$b^2 = 2c^2.$$

Now the same argument as above, but with b, a replaced by c, b, shows 2 divides b. Therefore 2 divides both a and b. But this contradicts that a, b had no common factors. □

§4.2, #18 Find the product of x and y supposing that both are odd.

This shows you an aligned string of equations.

$$\begin{aligned} xy &= (2a+1)(2b+1) \\ &= (2a)(2b) + (2a)(1) + 1(2b) + 1(1) \\ &= 4ab + 2a + 2b + 1 \\ &= 2(2ab + a + b) + 1 \\ &= 2k + 1, \end{aligned}$$

This shows you an aligned string of equations with justifications.

$$\begin{aligned} xy &= (2a+1)(2b+1) && a,\ b \text{ integers, (by definition of odd)} \\ &= (2a)(2b) + (2a)(1) + 1(2b) + 1(1) && \text{(expanding binomial multiplication)} \\ &= 4ab + 2a + 2b + 1 && \text{(simplifying)} \\ &= 2(2ab + a + b) + 1 && \text{(factoring out a 2)} \\ &= 2k + 1, && (k = 2ab + a + b) \end{aligned}$$

§4.2, #19 Make a table of useful LaTeXsymbols.

Table of LaTeXSymbols

words	what you type into LaTeX	what appears in the PDF	example
is an element of	\in	$\in$	$x \in \mathbb{R}$
is not an element of	\not \in	$\not\in$	$x \not\in \mathbb{R}$
is a subset of	\subseteq	$\subseteq$	$\mathbb{Q} \in \mathbb{R}$
curly brackets	\{ or \}	{ or }	
power set	\mathcal{ P }(A)	$\mathcal{P}(A)$	
intersection	\cap	$\cap$	
union	\cup	$\cup$	
implies	\Rightarrow	$\Rightarrow$	$P \Rightarrow Q$
for all	\forall	$\forall$	$\forall x \in \mathbb{R}$
there exists	\exists	$\exists$	
logical and	\land	$\land$	
logical or	\lor	$\lor$	
logical negation	\sim	$\sim$	
subscript	A_2	A_2	
superscript	xˆ2	x^2	
dollar sign	\$	$\$$	
number sign	\#	#	
ampersand	\&	&	
new line	\cr or \\		
backslash	\textbackslash	\	