Math 265
YOUR NAME HERE
March 19, 2026

Homework # 10

Problem List   Ch 10 # 4, 5, 8, 13, 18, 19, 22, A

Problem Directions: Prove using induction or strong induction.

4. If $n \in N$, then $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

5. If $n \in N$, then $2^1 + 2^2 + 3^3 + \cdots + 2^n = 2^{n+1} - 2$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

8. If $n \in \mathbb{N}$, then $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

13. Prove that $6 \mid (n^3 - n)$ for every integer $n \geq 0$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

18. Suppose $A_1, A_2, A_3, \cdots, A_n$ are sets in some universal set U and $n \geq 2$. Prove that

$$\overline{A_1 \cup A_2 \cup \cdots \cup A_n} = \overline{A_1} \cap \overline{A_2} \cap \cdots \cap \overline{A_n}.$$

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

19. Prove that $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} \leq 2 - \frac{1}{n}$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

22. If $n \in \mathbb{N}$, then $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{8}\right)\left(1 - \frac{1}{16}\right)\cdots\left(1 - \frac{1}{2^n}\right) \geq \frac{1}{4} + \frac{1}{2^{n+1}}$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

Problem A: Determine the set of positive integers n that can be written in the form $n = 4a + 5b$ where $a, b \in \mathbb{N} \cup \{0\}$. (Hint: Check the first few values of n directly, then use strong induction to show that, from a certain point n_0 onwards, all numbers n have such a representation.)

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

Proof by Induction Template

Proposition: For all $n \in \mathbb{N}$, the sum of the first n odd integers is n^2.

The proposition could be written as

$$\forall n \in \mathbb{N},\ 1 + 3 + 5 + \cdots + (2n-1) = n^2.$$

Proof. (by induction on n)
(1) **Base case ($n = 1$):** The left-hand side equals 1. The right-hand side equals $1^2 = 1$. Hence the statement holds for $n = 1$.

(2) **Inductive step:** Assume that for some $k \geq 1$,

$$1 + 3 + 5 + \cdots + (2k-1) = k^2.$$

We must show

$$1 + 3 + 5 + \cdots + (2k-1) + (2k+1) = (k+1)^2.$$

Starting from the left-hand side, observe

$$\begin{aligned}\big(1 + 3 + 5 + \cdots + (2k-1)\big) + (2k+1) &= k^2 + (2k+1) && \text{(inductive hypothesis)}\\ &= (k+1)^2. && \text{(factoring)}\end{aligned}$$

This completes the induction. □