Math 265
**YOUR NAME HERE**
March 25, 2026

# Homework # 11

Problem List
§11.1 # 1, 2, 6, 10, 15
§11.2 # 1, 2, 8, 13
§11.3 # 3, 8, 9
§11.4 # 3, 5, 6
§11.5 # 6, 7
§12.1 # 2, 3, 7, 8, 11

**11.1.1** Let $A = \{0, 1, 2, 3, 4, 5\}$. Write out the relation R that expresses $>$ on $A$.
**Answer:**

**11.1.2** Let $A = \{1, 2, 3, 4, 5, 6\}$. Write out the relation R that expresses $|$ (divides) on $A$.
**Answer:**

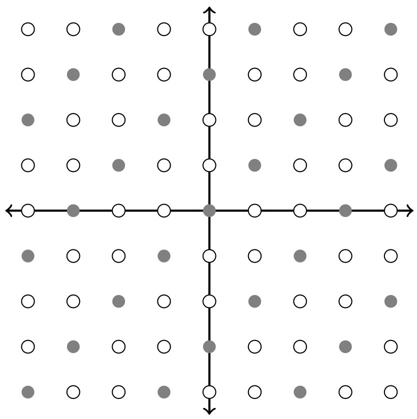
**11.1.6** Congruence modulo 5 is a relation on the set $A = \mathbb{Z}$. In this relation $x\,R\,y$ means

$$x \equiv y \pmod 5.$$

Write out the set R in set-builder notation.
**Answer:**

**11.1.10** Consider the subset $R = (\mathbb{R} \times \mathbb{R}) - \{(x, x) : x \in \mathbb{R}\} \subseteq \mathbb{R} \times \mathbb{R}$. What familiar relation on R is this? Explain.
**Answer:**

**11.1.15** A subset of $R$ of $\mathbb{Z}^2$ is indicated by gray shading. It is a familiar relation. State it.



**Answer:**

**11.2.1** Consider the relation $R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a)\}$ on set $A = \{a, b, c, d\}$. Is R reflexive? Symmetric? Transitive? If a property does not hold, say why.

**11.2.2** Consider the relation $R = \{(a, b), (a, c), (c, c), (b, b), (c, b), (b, c)\}$ on the set $A = \{a, b, c\}$. Is R reflexive? Symmetric? Transitive? If a property does not hold, say why.

**11.2.8** Define a relation on $\mathbb{Z}$ as $x\, R\, y$ if $|x - y| < 1$. Is R reflexive? Symmetric? Transitive? If a property does not hold, say why. What familiar relation is this?

**11.2.13** Consider the relation $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x - y \in \mathbb{Z}\}$ on R. Prove that this relation is reflexive, symmetric and transitive.

**11.3.3** Let $A = \{a, b, c, d, e\}$. Suppose R is an equivalence relation on A. Suppose R has three equivalence classes. Also $aRd$ and $bRc$. Write out R as a set and explicitly describe all three equivalence classes.

**11.3.8** Define a relation R on $\mathbb{Z}$ as $xRy$ if and only if $x^2 + y^2$ is even. Prove R is an equivalence relation. Describe its equivalence classes

**11.3.9** Define a relation R on Z as $xRy$ if and only if $4 \mid (x + 3y)$. Prove R is an equivalence relation. Describe its equivalence classes

**11.4.3** Describe the partition of $\mathbb{Z}$ resulting from the equivalence relation $\equiv \pmod 4$.

**11.4.5** Consider the partition $P = \big\{\{..., -4, -2, 0, 2, 4, ...\}, \{..., -5, -3, -1, 1, 3, 5, ...\}\big\}$ of $\mathbb{Z}$. Let R be the equivalence relation whose equivalence classes are the two elements of P. What familiar equivalence relation is R?

**11.4.6** Consider the partition $P = \big\{\{0\}, \{-1, 1\}, \{-2, 2\}, \{-4, 4\}, \ldots\big\}$ of $\mathbb{Z}$. Describe the equivalence relation whose equivalence classes are the elements of P.

**11.5.6** Suppose $[a], [b] \in \mathbb{Z}_6$ and $[a] \cdot [b] = [0]$. Is it necessarily true that either $[a] = [0]$ or$[b] = [0]$? What if $[a], [b] \in \mathbb{Z}_7$?

**11.5.7** Do the following calculations in $\mathbb{Z}_9$, in each case expressing your answer as $[a]$ with $0 \leq a \leq 8$.

(a) $[8] + [8]$

(b) $[24] + [11]$

(c) $[21] \cdot [15]$

(d) $[8] \cdot [8]$

**12.1.2** Suppose $A = \{a, b, c, d\}$, $B = \{2, 3, 4, 5, 6\}$ and $f = \{(a, 2), (b, 3), (c, 4), (d, 5)\}$. State the domain and range of $f$. Find $f(b)$ and $f(d)$.

**12.1.3** There are four different functions $f : \{a, b\} \to \{0, 1\}$. List them

**12.1.7** Consider the set $f = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : 3x + y = 4\}$. Is this a function from $\mathbb{Z}$ to $\mathbb{Z}$? Explain.

**12.1.8** Consider the set $f = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x + 3y = 4\}$. Is this a function from $\mathbb{Z}$ to $\mathbb{Z}$? Explain.

**12.1.11** Is the set $\theta = \{(X, |X|) : X \subseteq \mathbb{Z}_5\}$ a function? If so, what is its domain and range?