Math 265
YOUR NAME HERE
April 6, 2026

Homework # 12

Problem List
§12.2 # 8, 9, 10, 12, 14
§12.3 # 1, 2, 4, 7
§12.4 # 4, 6, 8

12.2.8 A function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ is defined as $f(m, n) = (m + n, 2m + n)$. Verify whether this function is injective and whether it is surjective.

Answer:

12.2.9 Prove that the function $f : \mathbb{R} - \{2\} \to \mathbb{R} - \{5\}$ defined by $f(x) = \frac{5x+1}{x-2}$ is bijective.

Answer:

12.2.10 Prove that the function $f : \mathbb{R} - \{1\} \to \mathbb{R} - \{1\}$ defined by $f(x) = \left(\frac{x+1}{x-1}\right)^3$ is bijective.

Answer:

12.2.12 Consider the function $\theta : \{0, 1\} \times \mathbb{N} \to \mathbb{Z}$ defined as $\theta(a, b) = a - 2ab + b$. Is θ injective? Is it surjective? Bijective? Explain

Answer:

12.2.14 Consider the function $\theta : \mathcal{P}(\mathbb{Z}) \to \mathcal{P}(\mathbb{Z})$ defined as $\theta(X) = \overline{X}$. Is θ injective? Surjective? Bijective? Explain.

Answer:

12.3.1 Prove that when at least six integers are chosen at random, then at least two of them will have the same remainder when divided by 5.

Answer:

12.3.2 Prove that if a is a natural number, then there exist two unequal natural numbers k and ℓ for which $a^k - a^\ell$ is divisible by 10.

Answer:

12.3.4 Consider a square whose side length is one unit. Select any five points from inside this square. Prove that at least two of these points are within $\sqrt{2}/2$ units of each other.

Answer:

12.3.7 (modified) Let $X \subseteq \{1, 2, 3, ..., 2n\}$ with $|X| > n$.

a. Prove that every positive integer m can be write in the form $2^i q$ where q is an odd integer.

b. Partition the elements of the set $\{1, 2, 3, \cdots, 14\}$ according to the relation mRn if m and n have the same q when written in the form $2^i q$, where q is odd.

c. Pick two different 8-element subsets of the set $\{1, 2, 3, \cdots, 14\}$. Try as much as possible to pick the 8 elements randomly. Show that each of the two subsets contain a pair of of distinct elements, a and b, where $a|b$.

d. Let $X \subseteq \{1, 2, 3, ..., 2n\}$ with $|X| > n$. Prove X contains two (unequal) elements for which one divides the other.

Answer:

12.4.4 Suppose $A = \{a, b, c\}$. Let $f : A \to A$ be the function $f = \{(a, c), (b, c), (c, c)\}$, and let $g : A \to A$ be the function $g = \{(a, a), (b, b), (c, a)\}$. Find $g \circ f$ and $f \circ g$.

Answer:

2.4.6 Consider the functions $f, g : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \frac{1}{x^2+1}$ and $g(x) = 3x + 2$. Find the formulas for $g \circ f$ and $f \circ g$.

Answer:

12.4.8 Consider the functions $f, g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined as $f(m, n) = (3m - 4n, 2m + n)$ and $g(m, n) = (5m + n, m)$. Find the formulas for $g \circ f$ and $f \circ g$.

Answer: