Math 265
YOUR NAME HERE
April 20, 2026

Homework # 13

Problem List

§12.5 # 2, 6, 8, 9
§12.6 # 2, 5, 6, 7, 8
§14.1 # 2, 8, 10, 11, 14
§14.2 # 2, 4, 5, 7, 8, 11, 12

12.5.2 In Exercise 9 of Section 12.2 you proved that $f : \mathbb{R} - \{2\} \to \mathbb{R} - \{5\}$ defined by $f(x) = \frac{5x+1}{x-2}$ is bijective. Now find its inverse.

Answer:

12.5.6 The function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined by the formula $f(m, n) = (5m + 4n, 4m + 3n)$ is bijective. Find its inverse.

Answer:

12.5.8 Is the function $\theta : \mathcal{P}(\mathbb{Z}) \to \mathcal{P}(\mathbb{Z})$ defined as $\theta(X) = \overline{X}$ bijective? If so, find θ^{-1}.

Answer:

12.5.9 Consider the function $f : \mathbb{R} \times \mathbb{N} \to \mathbb{N} \times \mathbb{R}$ defined as $f(x, y) = (y, 3xy)$. Check that this is bijective; find its inverse.

Answer:

12.6.2 Consider the function $f : \{1, 2, 3, 4, 5, 6, 7\} \to \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ given as $f = \{(1, 3), (2, 8), (3, 3), (4, 1), (5, 2), (6, 4), (7, 6)\}$. Find

a. $f(\{1, 2, 3\})$
Answer:

b. $f(\{4, 5, 6, 7\}$
Answer:

c. $f(\emptyset)$
Answer:

d. $f^{-1}(\{0, 5, 9\})$
Answer:

e. $f^{-1}(\{0, 3, 5, 9\})$
Answer:

12.6.5 Consider a function $f : A \to B$ and a subset $X \subseteq A$. We observed in Example 12.14 that $f^{-1}(f(X)) \neq X$ in general. However $X \subseteq f^{-1}(f(X))$ is always true. Prove this.

Answer:

12.6.6 Given a function $f : A \to B$ and a subset $Y \subseteq B$, is $f(f(Y)) = Y$ always true? Prove or give a counterexample.

Answer:

12.6.7 Given a function $f : A \to B$ and subsets $W, X \subseteq A$, prove $f(W \cap X) \subseteq f(W) \cap f(X)$.

Answer:

12.6.8 Given a function $f : A \to B$ and subsets $W, X \subseteq A$, then $f(W \cap X) = f(W) \cap f(X)$ is false in general. Produce a counterexample.

Answer:

Directions for §14.1 problems: Show that the two given sets have equal cardinality by describing a bijection from one to the other. Describe your bijection with a formula (not as a table).

14.1.2 $\mathbb{R}$ and $(\sqrt{2}, \infty)$

Answer:

14.1.8 $\mathbb{Z}$ and $S = \{x \in \mathbb{R} : \sin(x) = 1\}$

Answer:

14.1.10 $\{0, 1\} \times \mathbb{N}$ and $\mathbb{Z}$

Answer:

14.1.11 $[0, 1]$ and $(0, 1)$

Comment: This is very tricky. I gave a hint at the end of the homework. Also, we will learn an easier way to handle this in the future.

Answer:

14.1.14 $\mathbb{N} \times \mathbb{N}$ and $\{(n, m) \in \mathbb{N} \times \mathbb{N} : n \leq m\}$

Hint: I think it's easier to find a bijection from $\mathbb{N} \times \mathbb{N}$ to $\{(n, m) \in \mathbb{N} \times \mathbb{N} : n \leq m\}$. One way to help ensure the injectivity is to keep one coordinate the same.

Answer:

Directions: For the problems in section 14.2, you may use any theorem *from this section.* You must reference it the theorem explicitly as in "Applying Theorem 14.5 that says....., we can conclude...."

14.2.2 Prove that the set $A = \{(m, n) \in \mathbb{N} \times \mathbb{N} : m \leq n\}$ is countably infinite.

Answer:

14.2.4 Prove that the set of all irrational numbers is uncountable. (Suggestion: Consider proof by contradiction using Theorems 14.4 and 14.6.)

Answer:

14.2.5 Prove or disprove: There exists a countably infinite subset of the set of irrational numbers.

Answer:

14.2.7 Prove or disprove: The set $\mathbb{Q}^{100}$ is countably infinite.

Answer:

14.2.8 Prove or disprove: The set $\mathbb{Z} \times \mathbb{Q}$ is countably infinite.

Answer:

14.2.11 Describe a partition of $\mathbb{N}$ that divides $\mathbb{N}$ into eight countably infinite subsets.

Answer:

14.2.12 Describe a partition of $\mathbb{N}$ that divides $\mathbb{N}$ into $\aleph_0$ countably infinite subsets.

Answer:

Hint to 14.1.11 One way to think about finding a bijection $f : (0, 1) \to [0, 1]$ is that we just need to fill two "holes" at $x = 0$ and at $x = 1$. But if I define $f(1/2) = 1$, how do we replace it?

Consider the infinite sets $S = \{1/2, 1/4, 1/16, 1/32, \cdots\}$ and $T = \{1/4, 1/16, 1/32, \cdots\}$. How could you show that $|S| = |T|$? How can you use this idea to fill the hole at $x = 1/2$?