Math 265
April 21, 2026

Homework # 14

These do not need to be turned in.

Suggested Problem List
§14.3 # 1,3,4,5,7,8
§14.4 #1,2,3

14.3.1 Suppose B is an uncountable set and A is a set. Given that there is a surjective function $f : A \to B$, what can be said about the cardinality of A?

Answer:

14.3.3 Prove or disprove: If A is uncountable, then $|A| = |\mathbb{R}|$

Answer:

14.3.4 Prove or disprove: If $A \subseteq B \subseteq C$ and A and C are countably infinite, then B is countably infinite

Answer:

14.3.5 Prove or disprove: The set $\{0, 1\} \times \mathbb{R}$ is uncountable.

Answer:

14.3.7 Prove or disprove: If $A \subseteq B$ and A is countably infinite and B is uncountable, then $B - A$ is uncountable.

Answer:

14.3.8 Prove or disprove: The set $\{(a_1, a_2, a_3, ...) : a_i \in \mathbb{Z}\}$ of infinite sequences of integers is countably infinite.

Answer:

14.4.1 Show that if $A \subseteq B$ and there is an injection $g : B \to A$, then $|A| = |B|$.

Answer:

14.4.2 Show $|(0, 1)| = |(0, 1) \times (0, 1)|$. What does this say about $|\mathbb{R} \times \mathbb{R}|$?

Answer:

14.4.3 Let $\mathcal{F}$ be the set of all functions $\mathbb{N} \to \{0, 1\}$ Show that $|\mathbb{R}| = |\mathcal{F}|$.

Answer: