Homework 4

Section 2.7 Quantifiers: #1-10

Section 2.9 # 1-7, 9,10

Section 2.10 # 1-6, 8-10

Section 2.11 # A, B, C, D

Directions: For several problems you are asked to **write an argument in English.** In all cases, you are expected to use only complete sentences. You should use correct grammar and punctuation. Moreover, every sentence must begin with an English word and cannot begin with a symbol. So, a sentence that begins, "There exists ..." is OK. A sentence that begins "$\exists$..." is not. A sentence that begins "Observe the function $f(x) > 0$,..." is OK. A sentence that begins "$f(x) > 0$..." is not.

A. Use a truth table to show that the argument below is valid.

$$\begin{array}{l} P \Rightarrow Q \\ Q \Rightarrow R \\ \hline P \Rightarrow R \end{array}$$

B. i. Show that the argument below is an invalid argument by finding truth values for P and Q that demonstrate this.

$$\begin{array}{l} P \vee Q \\ P \\ \hline \sim Q \end{array}$$

ii. Write an argument in English with the logical structure in part (i.) and then show that the conclusion is false.
Example English Argument (that you cannot use): Every student in Introduction to Proofs has also passed Calculus 2 or Linear Algebra. Jane Doe passed Calculus 2. Thus, Jane Doe did not pass Linear Algebra.

C. i. Construct an **open, conditional statement**, $P(x) \Rightarrow Q(x)$, that you know to be true. This statement should be written as a sentence in English, but its content should be mathematical.
Example (that you cannot use): If $f(x)$ is differentiable on $\mathbb{R}$, then $f(x)$ is continuous on $\mathbb{R}$.

ii. Using your statement from part (i.), make an argument in English using Modus Ponens.
Example: If $f(x)$ is differentiable on $\mathbb{R}$, then $f(x)$ is continuous on $\mathbb{R}$. We know the function $f(x) = x^2$ is differentiable on $\mathbb{R}$ since its derivative is $f'(x) = 2x$ is defined on all of $\mathbb{R}$. Thus, we an conclude that the function $f(x) = x^2$ is continuous on $\mathbb{R}$.

iii. Using your statement from part (i.), make an argument in English using Modus Tollens.
Example: If $f(x)$ is differentiable on $\mathbb{R}$, then $f(x)$ is continuous on $\mathbb{R}$. We know the function $h(x) = \lfloor x \rfloor$ fails to be continuous at every integer. (FYI $h(x)$ is called the **floor** function or the "always round down" function. Its graph looks like a "stair steps.") Thus, we can conclude that $h(x)$ is not differentiable on $\mathbb{R}$.

D. Make an argument in English that the number 20 is composite using the logical structure of Elimination and exactly two facts.
Fact 1: Every integer larger than 1 is either prime or composite.
Fact 2: An integer larger than 1 is prime if its only divisors are 1 and itself.
Observe that the definition of **composite** is not given.