Math 265

YOUR NAME

February 12, 2026

Homework # 6

Due: Wednesday 02/18/2026

Problem List   Ch 4 #4,6,11,12,13,16,18,20,21,26,28

Problem Directions   Prove each statement below using a direct proof.

4. Suppose $x, y \in \mathbb{Z}$. If x and y are odd, then xy is odd.

Proof. YOUR PROOF GOES HERE □

Your comments on your own proof here. You can (should) include things like:

- How much help did you need? None? A little? A lot?
- Ask me questions. " I never did see where I used the hypothesis.... but it still seems right to me." or "My proof is so different from yours I can't tell if this is ok??"
- Corrections to your own work: I forgot to be careful about dividing by zero.

6. Suppose $a, b, c \in \mathbb{Z}$. if $a|b$ and $a|c$, then $a|(b + c)$.

Proof. YOUR PROOF GOES HERE □

Your comments on your own proof here.

11. Suppose $a, b, c, d \in \mathbb{Z}$. If $a|b$ and $c|d$, then $ac \mid bd$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

12. If $x \in \mathbb{R}$ and $0 < x < 4$, then $\frac{4}{x(4-x)} \geq 1$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

13. Suppose $x, y \in \mathbb{R}$. If $x^2 + 5y = y^2 + 5x$, then $x = y$ or $x + y = 5$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

16. If two integers have the same parity, then their sum is even.

Proof. YOUR PROOF GOES HERE □

Your comments on your own proof here.

18. Suppose x and y are positive real numbers. If $x < y$, then $x^2 < y^2$.

Proof. YOUR PROOF GOES HERE □

Your comments on your own proof here.

20. If a is an integer and $a^2 \mid a$, then $a \in \{-1, 0, 1\}$.

Proof. YOUR PROOF GOES HERE □

Your comments on your own proof here.

21. If p is prime and k is an integer for which $0 < k < p$, then p divides $\binom{p}{k}$.

Quick Review: For positive integers n and k, the symbol $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ and it counts the number of k-element subsets from a set with n-elements. By implication, the output of $\binom{n}{k}$ must be an integer. You can delete this review from your solutions.

Fact you can use: You can use the notion of a **prime factorization** of a positive integer – more specifically – any positive integer can be written as a product of prime numbers.

Proof. YOUR PROOF GOES HERE □

Your comments on your own proof here.

26. Every odd integer is the difference of two squares. (Example: $7 = 4^2 - 3^2$.)

Proof. YOUR PROOF GOES HERE □

Your comments on your own proof here.

28. Let $a, b, c \in \mathbb{Z}$. Suppose a and b are not both zero and $c \neq 0$. Prove that $c \cdot \gcd(a, b) \leq \gcd(ca, cb)$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.