Math 265

YOUR NAME

February 19, 2026

Homework # 7

Due: Wednesday 02/25/2026

$\boxed{\text{LaTeX Comments:}}$ In the preamble of the latex file you will see \newcommand. This is a way to make your own special short-cuts. For example, I got tired of typing: $\boxed{\texttt{\textbackslash textbf\{Z\}}}$. Thanks to a \newcommand, we can now type $\boxed{\texttt{\textbackslash Z}}$.

Even better, instead of typing $\boxed{\texttt{a \textbackslash equiv b \textbackslash :(\textbackslash text\{mod \} n)}}$ to get $\boxed{a \equiv b\,(\text{mod } n)}$, we can now type $\boxed{\texttt{\textbackslash modn\{a\}\{b\}\{n\}}}$

But you must have these "newcommands" in your preamble or your file will not compile.

$\boxed{\text{Problem List}}$ Ch 5 #4,6,7,10,11,18,19,24,25,27,28,30; Ch 6 # 4,8,9,10,12,14,15,16,18,22,23

$\boxed{\text{Problem Directions}}$

- Ch 5 #4-11: Use proof by contrapositive.
- Ch 5 #18-30: Use a direct proof or proof by contrapositive; it's your choice.
- Ch 6 # 4-18: Use proof by contradiction.
- Ch 6 # 22-23: Use any method. Note that we worked #20 in class.

Chapter 5

4. Suppose $a, b, c \in \mathbb{Z}$. If a does not divide bc, then a does not divide b.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

6. Suppose $x \in \mathbb{R}$. If $x^3 - x > 0$, then $x > -1$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

7. Suppose $a, b \in \mathbb{Z}$. If ab and $a + b$ are even, then both a and b are even.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

10. Suppose $x, y, z \in \mathbb{Z}$ and $x \neq 0$. If $x \nmid yz$, then $x \nmid y$ and $x \nmid z$.

Proof. YOUR PROOF GOES HERE □

Your comments on your own proof here.

11. Suppose $x, y \in \mathbb{Z}$. If $x^2(y+3)$ is even, then x is even or y is odd.

Proof. YOUR PROOF GOES HERE □

Your comments on your own proof here.

18. If $a, b \in \mathbb{Z}$, then $(a+b)^3 \equiv a^3 + b^3 \pmod 3$.

Proof. YOUR PROOF GOES HERE □

Your comments on your own proof here.

19. If $a, b, c \in \mathbb{Z}$ and $n \in \mathbb{N}$. If $a \equiv b \pmod{n}$ and $a \equiv c \pmod{n}$, then $c \equiv b \pmod{n}$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

24. If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $ac \equiv bd \pmod{n}$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

25. If $n \in \mathbb{N}$ and $2^n - 1$ is prime, then n is prime.

Recall the Calc 2 Review from Class: $r^n - 1 = (r-1)(r^{n-1} + r^{n-2} + \cdots + r + 1)$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

27. If $a \equiv 0 \pmod 4$ or $a \equiv 1 \pmod 4$, then $\binom{a}{2}$ is even.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

28. If $n \in \mathbb{Z}$, then $4 \nmid (n^2 - 3)$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

30. If $a \equiv b \pmod n$, then $\gcd(a, n) = \gcd(b, n)$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

Chapter 6

4. Prove $\sqrt{6}$ is irrational.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

8. Suppose $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then a or b is even.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

9. Suppose $a, b \in \mathbb{R}$. If a is rational and ab is irrational, then b is irrational.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

10. There exist no integers a and b for which $21a + 30b = 1$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

12. For every positive $x \in \mathbb{Q}$, there is a positive $y \in \mathbb{Q}$ for which $y < x$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

14. If A and B are sets, then $A \cap (B - A) = \emptyset$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

15. If $b \in \mathbb{Z}$ and $b \nmid k$ for every $k \in \mathbb{N}$, then $b = 0$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

16. If a and b are positive real numbers, then $a + b \geq 2\sqrt{ab}$.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

18. Suppose $a, b \in \mathbb{Z}$. If $4 \mid (a^2 + b^2)$, then a and b are not both odd.

Proof. YOUR PROOF GOES HERE $\square$

Your comments on your own proof here.

22. We showed in class that $x^2 + y^2 - 3 = 0$ can have no rational solutions. Use this fact to show that $x^2 + y^2 - 3^k = 0$ can have no rational solutions if k is an odd positive integer.

Proof. YOUR PROOF GOES HERE □

Your comments on your own proof here.

23. Use problem 22 (above) to prove that $\sqrt{3^k}$ is irrational for all odd, positive k.

Proof. YOUR PROOF GOES HERE □

Your comments on your own proof here.