1. Prove there are an infinite number of primes.

2. Prove that any two rational numbers x and y can be written in the form $x = \frac{a}{n}$ and $y = \frac{b}{n}$ such that $\gcd(a,b,n) = 1$ but that it is not possible to conclude that $\gcd(a,n) = 1$ or $\gcd(b,n) = 1$.

3. Statements about even and odd numbers could be rewritten in the language of integers modulo 2. For example, the statement:
   If n is odd, then n^2 is odd could be rewritten as If $n \equiv 1 \pmod{2}$, then $n^2 \equiv 1 \pmod{2}$.

   Make **conjectures** about what happens when you consider the squares of integers modulo 3 and then prove that you are correct.

4. Make a conjecture about when the sum of two integers can be congruent to 0 modulo 3 and prove that you are correct. Your proposition will look something like the one below:

   Proposition: Let $a, b \in \mathbb{Z}$ such that $a + b \equiv 0 \pmod{3}$, then [*something here about the nature of a and b modulo 3*].

5. Describe the set of points in the xy-plane that satisfy $x^2 + y^2 - 3 = 0$.

6. Prove that $x^2 + y^2 - 3 = 0$ contains no rational points. (That is, for every $(x, y) \in \mathbb{Q} \times \mathbb{Q}$, $x^2 + y^2 - 3 \neq 0$. Also, the previous propositions should help and please pause to think at least momentarily about how interesting this result is.)