

Below are four proofs of the assertion: For every pair of sets A and B , if $A \subseteq B$, then $A - B = \emptyset$.

Proof 1:

Suppose A and B are sets and $A \subseteq B$. So, every element of A is an element of B . So, no element can be in $A - B$. So, $A - B = \emptyset$.

Proof 2:

Suppose $A \subseteq B$. Recall that $A - B$ is the set of all elements that are in A but not in B . Since every element of A is also in B , there are no elements that are in A but not in B .

Now suppose for contradiction that $A - B \neq \emptyset$. Then there exists some element $x \in A - B$, which means $x \in A$ and $x \notin B$. But since $A \subseteq B$, we have $x \in B$.

Therefore $A - B = \emptyset$.

Proof 3:

(by contrapositive) Suppose $A - B \neq \emptyset$. Thus, there exists some $x \in A - B$. Since $x \in A - B$, it follows that $x \in A$ and $x \notin B$. Thus, it is not the case that every element of A is automatically an element of B . Thus, $A \not\subseteq B$.

Proof 4:

(by contrapositive) Suppose $A - B \neq \emptyset$. Thus, there exists some $x \in A - B$. Thus, $x \in A$ and $x \notin B$. Thus, $A \not\subseteq B$.