Introduction to Sets & Cartesian Products

1. A set is a collection of things, such that what is in the set and what is not in is well-defined.

   The things in the set are called elements.

   The number of elements in a set is its cardinality

2. Some examples and typical notation

   - capital letters for sets
   - $\{\}$ indicates a set.
   - "$\in$" = is an element of
   - $|A|$ denotes cardinality

   $A = \{1,2,3\}$, $1 \in A$, $4 \notin A$, $|A| = 3$

   $= \{2,1,3\}$

   R is the set of FBKs restaurants we have eaten at

   $B = \{ n \in \mathbb{N} : n \text{ is prime} \}$ ← set builder notation

   ↑ "such that"

   Natural numbers: 1, 2, 3, 4, ..

   $C = \{ 2, 3, \{0,1,2\}, f(x) = x^2 \}$; $\{\} = \emptyset$ = the empty set

3. The **Cartesian Product** of two sets A and B is

   $$A \times B = \{ (a,b) : a \in A, b \in B \}$$ ordered pair.

   $A \times B = \{(1,2),(1,3),(1,5),(1,7),(1,11),\ldots, (2,2),(2,3),(2,5)\ldots (3,2),(3,3),(3,5)\ldots\}$

   $\neq B \times A$

4. Some examples and typical notation

   $A \times A = \{ (1,1), (1,2), (1,3), (2,1) (2,2) (2,3) (3,1), (3,2), (3,3) \}$

   $= A^2$

   $|A \times A| = 3 \cdot 3 = 9$

   A^5 - 5-tuples w/entries in $\{1,2,3\}$.

   Sample element: $(1, 3, 1, 2, 2)$

   $|A^5| = 3^5$

   $|A \times B| = |A| \cdot |B|$

5. List the **elements** in each set below and determine its **cardinality**.

   (a) $A = \{1, 2, \{a, b, c\}, \emptyset\}$

   $1, 2, \{a, b, c\}, \emptyset$

   $|A| = 4$

   (b) $A = \{x \in R : x^3 - x^2 = 6x\}$ $= \{0, 3, -2\}$

   $x^3 - x^2 - 6x = 0$

   $x(x^2 - x - 6) = x(x-3)(x+2)$ $\quad |A| = 3$

   (c) $A = \{x \in \mathbb{N} : x^3 - x^2 = 6x\} = \{3\}$ $\quad |A| = 1$

6. Write each set in **set-builder** notation.

   (a) The half-open interval of the real line: $[2, 8)$.

   $[2,8) = \{x \in R : 2 \leq x < 8\}$

   (b) $\{-6, -3, 0, 3, 6, 9, 12, \ldots\}$ (Assume the pattern continues.)

   $\{3n : n \in \mathbb{Z} \text{ and } n \geq -2\}$

   $\{n \in \mathbb{Z} : 3 | n \text{ and } n \geq -6\}$

   (c) The set of points in the xy-plane that lie on the graph $y = x^2 + 1$.

   $\{(x, y) \in \mathbb{R} \times \mathbb{R} : y = x^2 + 1\}$

7. Sketch $[0, 1]^3$ ← the unit cube