

1. Quick Review

(a) R is a relation on A if

(b) Suppose R is a relation on the set A .

i. We say R is **symmetric** if

ii. We say R is **reflexive** if

iii. We say R is **transitive** if

(c) What properties do the following relations have?

i. Let R be a relation on $\mathbb{N} \times \mathbb{N}$ such that $(a,b)R(c,d)$ if $ad = bc$.

ii. Let R be a relation on $A = \mathbb{Z}$ defined by $a R b$ if $a \equiv b \pmod{3}$.

iii. Suppose R is a relation on $\mathcal{P}(A)$ where $A = \{0, 1, 2, 3, 4\}$ defined as $X R Y$ if $X \cap Y \neq \emptyset$.

2. A relation R on set A is an **equivalence relation** if

3. Build your own equivalence relation on $A = \{a, b, 1, 5\}$.

4. For each relation below, quickly confirm the relation **is** an equivalence relation, then identify its equivalence classes using the square bracket notation **in two different ways** and by describing them as sets. Can you describe the partition of A that is produced?

(a) Let R be a relation on $A = \mathbb{Z}$ defined by $a R b$ if $a \equiv b \pmod{6}$.

(b) Let R be a relation on the set of all polynomials with real coefficients defined by $p(x) R q(x)$ if the degree of $p(x)$ equals the degree of $q(x)$.

(c) Let R be the set of differentiable functions $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) R g(x)$ if $f'(x) = g'(x)$.

5. Suppose R and S are two equivalence relations on A . Answer the following questions **rigorously**.

(a) Is the set $R \cup S$ an equivalence relation on A ?

(b) Is the set $R \cap S$ an equivalence relation on A ?