

4. For each function below, determine if it is injective or surjective and prove your answer is correct. If the function is not surjective, determine its range.

(a) $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ defined as $f(m, n) = 2m - 3n$.

(b) $\theta : \{0, 1\} \times \mathbb{N} \rightarrow \mathbb{Z}$ defined as $\theta(a, b) = (-1)^a b$.

(c) $g : \mathbb{N} \rightarrow \mathbb{Z}$ defined as $g(n) = \begin{cases} \frac{n}{2} & n \text{ even} \\ (-1)^{\frac{n-1}{2}} & n \text{ odd} \end{cases}$