1. The Pigeonhole Principle

   (a) (version 1)

   (b) (version 2)

2. Given any 10-element subset of $\{1,2,3,\cdots,100\}$ there exist at least two distinct subsets with the same sum.

3. Let $B = \{\{0\},\{1,8\},\{2,7\},\{3,6\},\{4,5\}\}$ be a partition of $\{0,1,2,3,\cdots,9\}$.
   Suppose that $f(n) : \mathbb{N} \to B$ maps a natural number to the subset of $B$ that contains its remainder upon division by 9.

   (a) Determine $f(13)$ and $f(28)$.

   (b) Prove that if $f(n) = f(m)$, then either $9 \mid (n+m)$ or $9 \mid (n-m)$.

(c) Fill in the blank below with the **smallest** possible value for which it is true and then prove the statement using PHP v2.

Every subset of $\mathbb{N}$ with at least _____ elements will contain a pair of numbers whose sum or difference is divisible by 9.

4. Let $A = \{a,b,c\}, B = \{1,2,3\}$, and $C = \{4,5,6,7\}$.
Suppose $f : A \to B$, $g : B \to C$ and $h : C \to B$ are functions defined below.

$$f = \{(a,1),(b,2),(c,2)\}, \quad g = \{(1,4),(2,5),(3,6)\}, \quad h = \{(4,1),(5,2),(6,2),(7,3)\}.$$

Describe each expression below by using "potato" pictures, stating explicitly the associated ordered pairs, and establishing the domain, codomain and range. Note "$\circ$" represents composition of functions just like in precalculus:

$$g \circ f = g(f(x))$$

(a) $g \circ f$

(b) $(h \circ g) \circ f$