

1. Give a rationale and/or proof for the following statements.

(a) If  $A$  is countably infinite and  $B \subseteq A$ , then  $B$  is countable.

(b) If  $A$  and  $B$  are both countably infinite, then  $A \cup B$  is countably infinite.

(c) If  $A$  and  $B$  are both countably infinite, then  $A \times B$  is countably infinite.

- (d) If  $A_1, A_2, \dots, A_k$  is a set of  $k$  countably infinite sets, then
- i.  $A_1 \cup A_2 \cup \dots \cup A_k$  is countably infinite.

ii.  $A_1 \times A_2 \times \dots \times A_k$  is countably infinite.

2. Match the statements on the left with an equivalent statement on the right for sets  $A$  and  $B$  and function  $f : A \rightarrow B$ .

$ A  =  B $	$f$ is injective but not surjective
	$f$ is surjective but not injective
$ A  \leq  B $	$f$ is surjective and injective
	$f$ is neither surjective nor injective
$ A  <  B $	$f$ is injective
	$f$ is surjective