

1. Give a rationale and/or proof for the following statements.

(a) If A is countably infinite and $B \subseteq A$, then B is countable.

(See Thm 14.8 for a formal proof)

If B is finite, we're done

If B is infinite, then use Thm 14.3 that states that A can be written as an infinite list:

$$A = a_1, a_2, a_3, a_4, \dots$$

Since $B \subseteq A$, the elements of B can be listed too!
They are a sublist.

(b) If A and B are both countably infinite, then $A \cup B$ is countably infinite.

(See Thm 14.6 for a formal proof.)

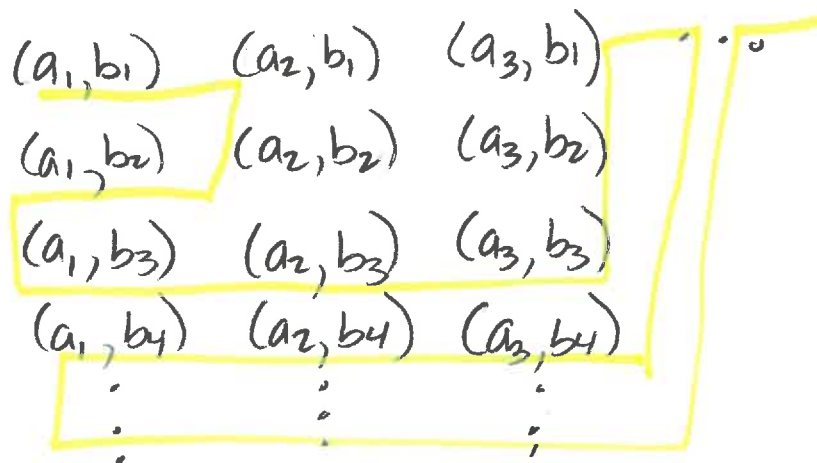
Thm 14.3 says $A : a_1, a_2, a_3, \dots$ and $B : b_1, b_2, b_3, \dots$

So $A \cup B : a_1, b_1, a_2, b_2, a_3, b_3, \dots$ or more formally

$$f: \mathbb{N} \rightarrow A \cup B \text{ defined as } f(n) = \begin{cases} a_{\frac{n+1}{2}} & n \text{ odd} \\ b_{\frac{n}{2}} & n \text{ even} \end{cases}$$

(c) If A and B are both countably infinite, then $A \times B$ is countably infinite.

Arrange $A \times B$ in a grid:



Proceed w/
winding path
like in proof
that $|\mathbb{Q}| = |\mathbb{N}|$.

See Thm 14.5 for
a formal proof

(d) If A_1, A_2, \dots, A_k is a set of k countably infinite sets, then

i. $A_1 \cup A_2 \cup \dots \cup A_k$ is countably infinite.

The argument here is the same as for (d)(ii) or Cor 14.1

ii. $A_1 \times A_2 \times \dots \times A_k$ is countably infinite.

(See Corollary 14.1 for formal proof)

$A_1 \times A_2$ is C.I. by Thm 14.5

Now $(A_1 \times A_2) \times A_3$ is C.I. by Thm 14.5

Now $(A_1 \times A_2 \times A_3) \times A_4$ is C.I. by Thm 14.5

\vdots

So $(A_1 \times A_2 \times \dots \times A_{k-1}) \times A_k$ is C.I. by Thm 14.5

This is induction on k . $\begin{pmatrix} \forall \forall \\ \exists \exists \end{pmatrix}$

2. Match the statements on the left with an equivalent statement on the right for sets A and B and function $f : A \rightarrow B$.

