Subsets, Power Set, Set Operations

1. Definitions

   (a) The set B is a **subset** of the set A if every element in B is also an element in A. Written $\boxed{B \subseteq A}$

Ex: $\{1,3\} \subseteq \{1,2,3\}$, $\{1,4\} \not\subseteq \{1,2,3\}$; $\mathbb{N} \subseteq \mathbb{R}$

   (b) The **power set** of a set A is the set of all subsets of A, Denoted $\mathcal{P}(A)$.

Ex $A = \{1,2\}$,

Not "1" or $\{\}$ but NOT $\{\emptyset\}$

$\mathcal{P}(A) = \{ \{1\}, \{2\}, \{1,2\}, \emptyset \}$

   (c) The **union** of sets A and B is the set of elements in A or in B.

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Ex $A = \{1,2\}$ $B = \{2,3\}$, $A \cup B = \{1,2,3\}$

   (d) The **intersection** of sets A and B is the set of elements in both A and B.

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

$A \cap B = \{2\}$, $A \cap A = \{1,2\} = A$, $A \cap \{5,6\} = \emptyset$

   (e) The **difference** of sets A and B, say $A - B$, is the set of elements in A and not in B.

$A - B = \{x : x \in A \text{ and } x \notin B\}$.

$A - B = \{1,2\} - \{2,3\} = \{1\}$

   (f) The **complement** of the set A is denoted $\overline{A}$ (elsewhere A^c) and requires a universal set U. $\overline{A} = \{x \in U : x \notin A\}$.

Say $U = \{1,2,3,4\}$, $\overline{A} = \{3,4\}$

2. Let $C = \{1,2,3\}$

   (a) List all the elements of the set C. 1, 2, 3

   (b) List all the subsets of the set C.

   $\mathcal{P}(C) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

   (c) Find $\mathscr{P}(C)$. (You have permission to answer this by small changes to your answer in (b).)

   (d) Find $\mathscr{P}(\emptyset)$. $= \{\emptyset\}$

3. Determine whether each statement below is true or false and be prepared to give a reason for your answer.

   $\{\{a\}\} \subseteq \{a,b,c,d,\{a\},\{a,e\}\}$ T

   T (a) $a \in \{a,b,c,d,\{a\},\{a,e\}\}$

   Not a set. Nonsense.

   F (b) $a \subseteq \{a,b,c,d,\{a\},\{a,e\}\}$

   T (c) $\{a\} \in \{a,b,c,d,\{a\},\{a,e\}\}$

   T (d) $\{a\} \subseteq \{a,b,c,d,\{a\},\{a,e\}\}$

   T (e) $\emptyset \subseteq \{a,b,c,d,\{a\},\{a,e\}\}$ always

   F (f) $\emptyset \in \{a,b,c,d,\{a\},\{a,e\}\}$ No "$\emptyset$" in here.

   F (g) $\{a,e\} \subseteq \{a,b,c,d,\{a\},\{a,e\}\}$ ← it's an element

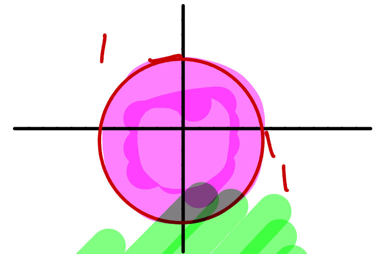
   T (h) $\{\{a,e\}\} \subseteq \{a,b,c,d,\{a\},\{a,e\}\}$

   F (i) $e \in \{a,b,c,d,\{a\},\{a,e\}\}$ ← No "e" by itself

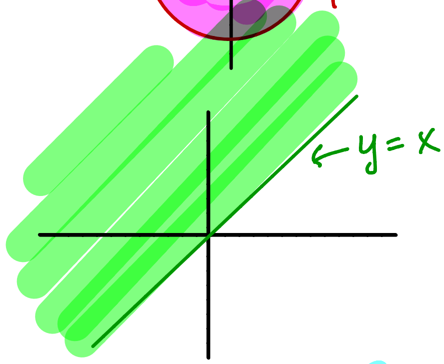
   F (j) $\{e\} \subseteq \{a,b,c,d,\{a\},\{a,e\}\}$

4. Let $A = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ and let $B = \{(x,y) \in \mathbb{R}^2 : x \leq y\}$. Suppose the universal set U is $\mathbb{R} \times \mathbb{R}$. Sketch each of the following on separate axes.

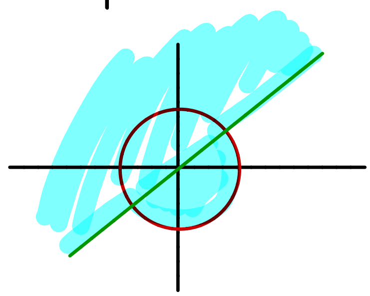
(a) A



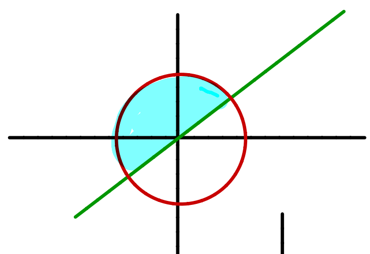
(b) B



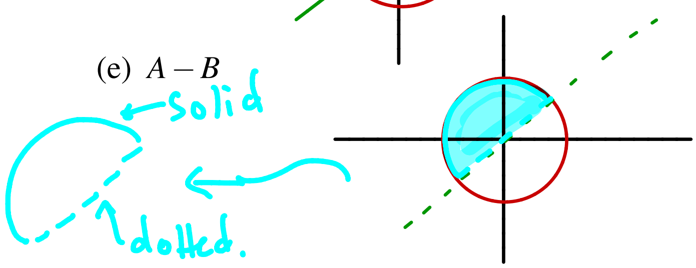
(c) $A \cup B$



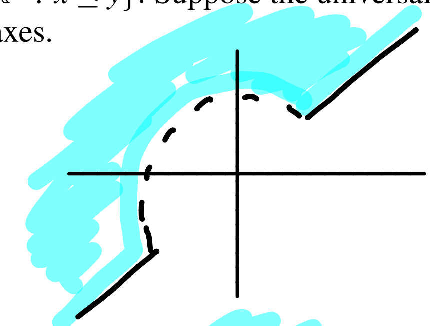
(d) $A \cap B$



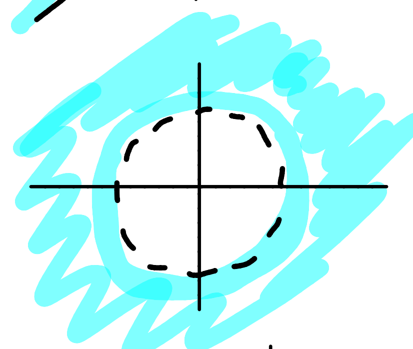
(e) $A - B$



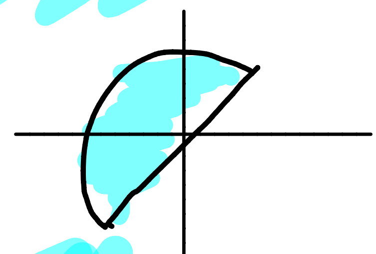
(f) $B - A$



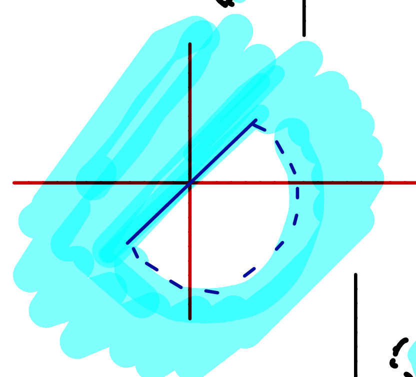
(g) $\overline{A}$



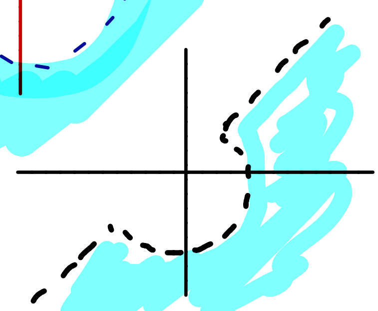
(h) $A - \overline{B}$



(i) $\overline{A - B}$



(j) $\overline{A \cup B}$



5. How many subsets does $D = \{1,2,3,4,5\}$ have? Justify your answer.

$2^5 = 32$. For each element, there are 2 choices: in or out.

6. Determine $|\mathscr{P}(A)|$ where A is a set with n elements.

$|\mathcal{P}(A)| = 2^n$