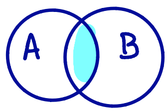
Venn Diagrams and Indexed Sets

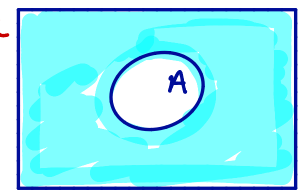
1. Venn Diagrams – a common cartoon for sets

Ex | Use a Venn diagram to show $A \cap B$.



Ex | Use Venn diag. to show $\overline{A}$.

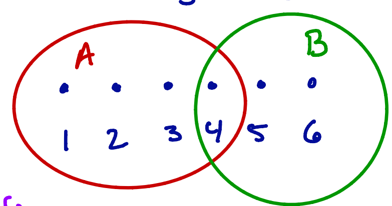
$\mathcal{U}$



← Need $\mathcal{U}$.

general →

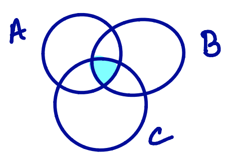
Idea Say $A = \{1,2,3,4\}$, $B = \{4,5,6\}$



$A \cap B = \{4\}$

↗ Specific

Ex | Use a Venn diag to show $A \cup B \cup C$.



2. Indexed Sets

(a) Finite Examples and Definitions

Ex | $A_1 = \{a, b\}$, $A_2 = \{a, c\}$, $A_3 = \{a, b, d\}$ ← A_1, A_2, A_3 forms a set of sets (!)

$$A_1 \cup A_2 \cup A_3 = \{a, b, c, d\} = \bigcup_{i=1}^{3} A_i$$

$$\bigcap_{i=1}^{3} A_i = A_1 \cap A_2 \cap A_3 = \{a\}$$

def: Sppse $A_1, A_2, \ldots, A_n$ are sets.

$$\bigcup_{i=1}^{n} A_i = \{x : x \in A_i \text{ for at least one } A_i,\ i \in \{1,2,3\}\}$$

$$\bigcap_{i=1}^{n} A_i = \{x : x \in A_i \text{ for every } A_i,\ i \in \{1,2,3\}\}$$

(b) Infinite and More General Examples and Definitions

Ex $A_1 = \{0,1\}$, $A_2 = \{0,1,2\}$, $A_3 = \{0,1,2,3\}, \dots$ infinite # of sets.

or $A_n = \{0,1,\dots,n\}$ for every $n \in \mathbb{N}$.

Expand our definition

$$\bigcup_{n=1}^{\infty} A_n = A_1 \cup A_2 \cup A_3 \cup \dots = \{x : x \in A_n \text{ for at least one } A_i \text{ with } i \in \mathbb{N}\}$$

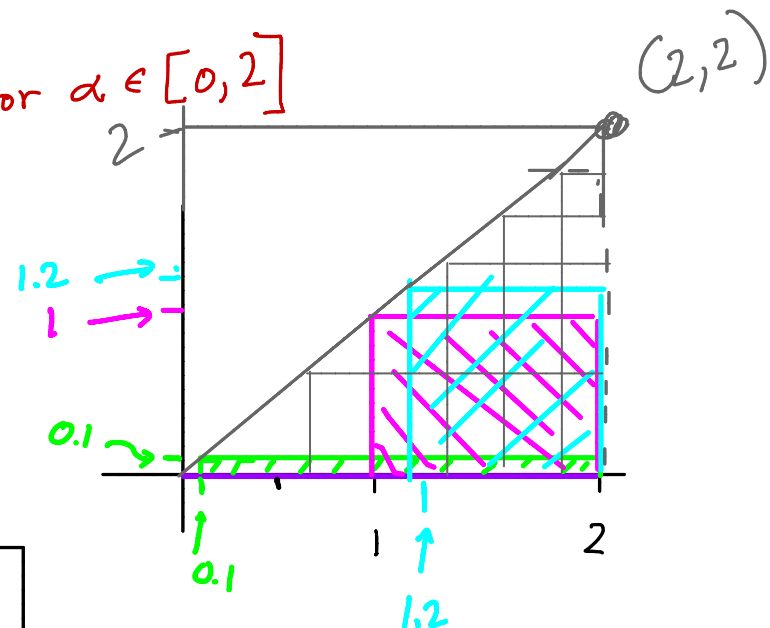
$$\bigcap_{n=1}^{\infty} A_n = A_1 \cap A_2 \cap A_3 \dots = \{x : x \in A_n \text{ for every } A_i,\ i \in \mathbb{N}\}$$

In this example: $\bigcup_{n=1}^{\infty} A_n = \mathbb{N} \cup \{0\}$; $\bigcap A_i = \{0,1\}$

Not all sets are countable!

Ex $A_\alpha = [\alpha, 2] \times [0, \alpha]$ for $\alpha \in [0,2]$

- $A_0 = [0,2] \times [0,0] = [0,2] \times \{0\}$
- $A_{0.1} = [0.1, 2] \times [0, 0.1]$
- $A_1 = [1,2] \times [0,1]$
- $A_{1.2} = [1.2, 2] \times [0, 1.2]$



Notation A_i sets indexed by I

$$\bigcup_{i \in I} A_i = \{x : x \in A_i \text{ for at least one } A_i,\ i \in I\}$$

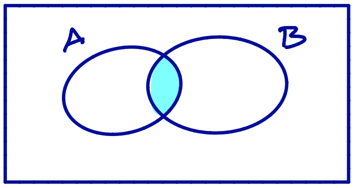
$$\bigcap_{i \in I} A_i = \{x : x \in A_i \text{ for every } A_i,\ i \in I\}$$

$$\bigcup_{\alpha \in [0,2]} A_\alpha = \{(x,y) : 0 \le x \le 2, 0 \le y \le x\}$$

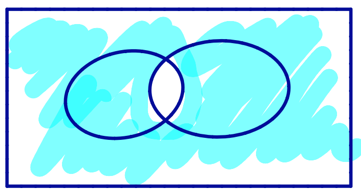
$$\bigcap A_\alpha = \{(2,0)\}$$

3. Draw a Venn Diagram for each set and then answer the questions.

(a) $A \cap B$

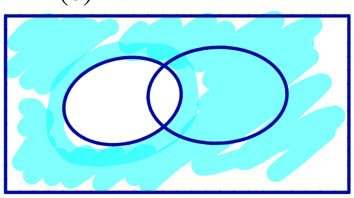


(b) $\overline{A \cap B}$

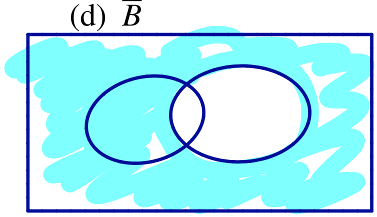


the same

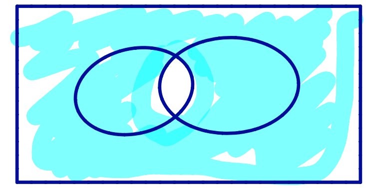
(c) $\overline{A}$



(d) $\overline{B}$



(e) $\overline{A} \cup \overline{B}$



(f) Use the work above to make a conjecture.

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

de Morgan's Laws

(g) Make a conjecture about $\overline{A \cup B}$ and check it with a Venn Diagram.

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

4. Suppose $A_n = \{n, n+1, \ldots, 2n\}$ for $n \in \mathbb{N}$.

(a) Determine the sets A_1, A_2, and A_3 by writing out their elements.

$A_1 = \{1, 2\}$, $A_2 = \{2, 3, 4\}$, $A_3 = \{3, 4, 5, 6\}$,

(b) $\bigcup_{n \in \mathbb{N}} A_n = \mathbb{N}$

(c) $\bigcap_{n \in \mathbb{N}} A_n = \emptyset$

5. Suppose $B_\alpha = [1, 3-\alpha] \subseteq \mathbb{R}$ for $\alpha \in [0,1]$.

   (a) Determine the sets B_α for four different values of α.

   $B_0 = [1,3]$ $B_{1/2} = [1,2.5]$ $B_{0.9} = [0,2.1]$

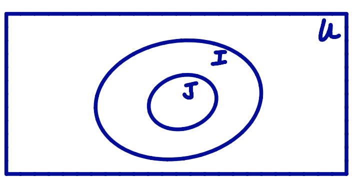
   $B_{0.1} = [1,2.9]$

   (b) $\bigcup_{\alpha\in[0,1]} A_\alpha = [1,3]$

   (c) $\bigcap_{\alpha\in[0,1]} A_\alpha = [1,2]$

6. For each $i \in I$, A_i is a set. Suppose $J \subseteq I$.

   (a) Draw a Venn diagram of sets I and J.



   (b) Is it possible to determine the relationship between:

      i. $\bigcup_{i\in I} A_i$ and $\bigcup_{j\in J} A_j$? Explain.

      $\bigcup_{j\in J} A_j \subseteq \bigcup_{i\in I} I_i$

      ii. $\bigcap_{i\in I} A_i$ and $\bigcap_{j\in J} A_j$? Explain.

      $\bigcap_{i\in I} A_i \subseteq \bigcap_{j\in J} A_j$.

Proof

We need to show that every element in $\bigcup_{j\in J} A_j$ is also an element in $\bigcup_{i\in I} A_i$.

Let $x \in \bigcup_{j\in J} A_j$. By the definition of set union, $x \in A_{j_0}$ some $j_0 \in J$. Since $J \subseteq I$ and $j_0 \in J$, it follows that $j_0 \in I$.

So $x \in A_{j_0}$ some $i \in I$. So $x \in \bigcup_{i=I} I_i$.