1. Suppose P and Q are true and R and S are false. Determine the truth value of each logical statement below. Think about how you can write down your reasoning or your work.

   (a) $(P \vee Q) \wedge (R \vee S)$

   $= (T \vee T) \wedge (F \vee F)$
   $= T \wedge F$
   $= F$

   (b) $(P \vee R) \Rightarrow (Q \wedge S)$

   $= (T \vee F) \Rightarrow (T \wedge F)$
   $= T \Rightarrow F$
   $= F$

   (c) $((P \wedge \sim P) \Rightarrow S) \Rightarrow Q$

   $= ((T \wedge F) \Rightarrow F) \Rightarrow T$
   $= (F \Rightarrow F) \Rightarrow T$
   $= T \Rightarrow T$
   $= T$

2. For each sentence below, write its logical structure in symbols. Make sure to clarify which words are associated with which letters. Hint: All but (b) and (c) are conditional statements.

   (a) Differentiability is sufficient for continuity.

   Differentiability implies continuity or
   $P \Rightarrow Q$ where P is differentiability and Q is continuity.

   Sufficient conditions are hypotheses.

   (b) At least one of a or b is an integer.

   $a \in \mathbb{Z}$ or $b \in \mathbb{Z}$

   $P \vee Q$ where P is $a \in \mathbb{Z}$ and Q is $b \in \mathbb{Z}$

   (c) Both A and B are subsets of C.
   $P \wedge Q$ where P is $A \subseteq C$ and Q is $B \subseteq C$.

(d) The grass is green whenever the sky is blue.

Equivalently: If the sky is blue, then the grass is green.

$P \Rightarrow Q$ where P is the sky is blue and Q is grass is green.

(e) My car turns on only if it's a leap year.

Equivalently: If my car turns on, then it's a leap year.
$P \Rightarrow Q$

(f) It is a Monday provided the door is open.

Equivalently: If the door is open, then it is a Monday. $P \Rightarrow Q$

(g) Warm bread is necessary for cold water.

Equivalently: If the water is cold then the bread is warm.
$P \Rightarrow Q$

Jill's Notes

The purpose of Chapter 2 is to rigorously define and understand the underlying logical structure of sentences and arguments written in words. We want to be able to:

- Go back and forth between English and logical symbols
- How to determine the truth value of very complicated statements
- How to determine whether an argument is logically sound and, if not, identify the error.

1. (2.1) A **statement** is an assertion (sentence, mathematical expression) that is true or false, typically denoted with capital letters, P, Q, R, etc.
   Some examples and non-examples below.
   (a) $2 \geq 1$ A statement that is true.
   (b) $f(x) = 1/x$ is continuous on $(-\infty, \infty)$ A statement that is false.
   (c) $ax^2 + bx + c$ Not a statement.
   (d) If $f(x)$ is differentiable, then $f(x)$ is continuous. A statement that is true.
   (e) $\mathbb{Z} \subseteq \mathbb{N}$. A statement that is false.
2. (2.2 and 2.3) New Statements from Old
   - Consider some ways to combine statements into more complicated statements.
   - Determine truth value of the combined statement based on the truth values of its (simpler, not compound) statements.
   - How to use a truth table to define/communicate truth values.
3. Some examples to think about intuitively. The goal here is to see that the definitions we are about to state are intuitive.
   (a) **OR**:
   P or Q
   $P \vee Q$
   Examples (Should R, S, T, U be true or false?)
   R: $\mathbb{Z} \subseteq \mathbb{N}$ or $\mathbb{N} \subseteq \mathbb{Z}$. (true)
   S: 2 is odd or 2 is negative. (false)
   T: $2 \in \mathbb{N}$ or $2 \in \mathbb{R}$. (true)
   U: Today I will ski or go to the gym.
   You are intuiting a definition.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

(b) **AND**: P and Q , $P \wedge Q$
Examples (Should R,S,T,U be true or false?)
R: $\mathbb{Z} \subseteq \mathbb{N}$ and $\mathbb{N} \subseteq \mathbb{Z}$. (false)
S: 2 is odd and 2 is negative. (false)
T: $2 \in \mathbb{N}$ and $2 \in \mathbb{R}$. (true)
U: Today I will ski and go to the gym.
You are intuiting a definition.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

(c) **NOT**: not P , $\sim P$
Examples (Should R,S,T,U be true or false?)
R: $\sim (\mathbb{N} \subseteq \mathbb{Z})$ which could also be $\mathbb{N} \not\subseteq \mathbb{Z}$ (false)
S: $\sim$(2 is odd) which could be written 2 is even (true)
U: ”It is not the case that today I will ski.” or, less awkwardly, “I will not ski today.”
You are intuiting a definition.

P	$\sim P$
T	F
F	T

Observe this means that if statement P is not true, it must be false; and if P is not false, it must be true.

(d) **Conditional**:
If P, then Q
$P \Rightarrow Q$
Examples (Should R,S,T be true or false?)
R: If $x \leq 5$, then $x \leq 10$. (true)
S: If $x \in \mathbb{R}$, then $x \leq x^2$. (false, take $x = 0.5$)
U: If I walk, then I’ll be late.
U’: If I have wings, then I’ll fly. (vacuously true)
T: If $\pi \in \mathbb{Z}$, then 2π is even. (vacuously true)
You are intuiting a definition.

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T