1. The Biconditional

   Suppose P and Q are statements. Then the statement P if and only if Q (symbolically $P \Leftrightarrow Q$ should be true if

   - (in words) P and Q have the same truth value – so both true or both false.
   - (truth table)

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

2. Examples:

   (a) R: $ab = 0$ if and only if $a = 0$ or $b = 0$ (true for real numbers)

   (b) S: f is differentiable if and only if f is continuous (false, in general)

3. Logical Equivalence

   Two (compound) statements are (logically) equivalent if they have the same truth value for all possible truth values of the "input" statements.

4. Claim: $\sim (P \vee Q)$ is equivalent to $\sim P \wedge \sim Q$

   Proof: Construct a truth table.

1 P	2 Q	3 $P \vee Q$	4 $\sim (P \vee Q)$	5 $\sim P$	6 $\sim Q$	7 $\sim P \wedge \sim Q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

   Since 4 and 7 have the same truth values, $\sim (P \vee Q)$ is equivalent to $\sim P \wedge \sim Q$.

   Think about what this means in words: The truth value of the three statements below are the same.

   (1) It is not the case that $x \geq 2$ or that $x \leq 5$.

   (2) $x \ngeq 2$ and $x \nleq 5$.

   (3) $x < 2$ and $x > 5$.

5. Claim: $P \vee (Q \wedge R)$ is **not** equivalent to $(P \vee Q) \wedge R$

   Option 1: Make a truth table and show that the columns are different in some particular row.

   Option 2: Find (via intuition or a truth table) some particular truth values for which the expressions are different.

   Proof: Suppose P is true but Q and R are both false. Then the statement $P \vee (Q \wedge R)$ is true because P is true. But the statement $(P \vee Q) \wedge R$ is false because R is false. Since the expressions do not have the same truth values, they cannot be logically equivalent.