1. Review

   (a) $\sim \big(\forall n \in \mathbb{N},\, 2n^2 - n \geq 1\big)$

   (b) $\sim \big(\exists n \in \mathbb{N},\, 2n^2 - n > 10\big)$

2. From the previous sheet:

   d. There are squares with integer values for the sides and the diagonals.

   $$\exists \text{ squares with side } s \text{ and diagonal } d,\, \big(s \in \mathbb{Z} \wedge d \in \mathbb{Z}\big)$$

   We concluded it was false. What do we need to show?

   e. Every integer that is not positive must be negative.

   $$\forall n \in \mathbb{N}, \sim \big(n > 0\big) \Rightarrow \big(n < 0\big)$$

   We concluded it was false. What do we need to show?

   g. For every quadratic polynomial $p(x)$, there is some real number a, where a is a root of $p(x)$.

   $$\forall p(x) \in \mathbb{P}_2(x),\, \exists a \in \mathbb{R},\, p(a) = 0, \quad \text{where } \mathbb{P}_2(x) \text{ is the set of degree 2 polynomials}$$

   We concluded it was false. What do we need to show?

3. Logical Inference:

4. Modus Ponens

5. The most common fallacy (invalid argument):

6. For each argument below, (a) determine whether it is valid or invalid, (b) write an argument in English that models the logical structure of the argument.

(a)
$$\begin{array}{l} P \Rightarrow Q \\ \sim P \\ \hline \sim Q \end{array}$$

(b)
$$\begin{array}{l} P \Rightarrow Q \\ \sim Q \\ \hline \sim P \end{array}$$

(c)
$$\begin{array}{l} P \vee Q \\ \sim P \\ \hline Q \end{array}$$

7. Show that $\boxed{P \Rightarrow Q}$ is logically equivalent to $\boxed{\sim Q \Rightarrow \sim P.}$ (Note: $\sim Q \Rightarrow \sim P$ is called the **contrapositive** of $P \Rightarrow Q$.)

8. Rewrite each theorem below with its equivalent contrapositive statement. Note that the "Let..." sentence does not change.

   (a) If two sides of a triangle are congruent (aka of equal length), then the two angles opposite those sides are congruent (aka are equal in measure).

   (b) Let $f(x)$ be defined on the interval $[a,b]$. If $f(x)$ is continuous on $[a,b]$, then for every y-value, y_0, strictly between $f(a)$ and $f(b)$ there exists an x-value, x_0, in (a,b) such that $f(x_0) = y_0$.