1. Definitions and Facts

   (a) An integer n is **even** if

   (b) An integer n is **odd** if

   (c) Let $a, b \in \mathbb{Z}$. We say a **divides** b if

   Alternate wording:

   (d) A number $n \in \mathbb{N}$ is **prime** if

   A number $n \in \mathbb{N}$ is **composite** if

   (e) Let $a, b \in \mathbb{Z}$. The **greatest common division of** a **and** b (denoted $\gcd(a,b)$) is

   (f) $a, b \in \mathbb{Z} - \{0\}$. The **least common multiple of** a **and** b (denoted $\operatorname{lcm}(a,b)$) is

   (g) **Fact 4.1:** If $a, b, \in \mathbb{Z}$, then $a+b$, $a-b$, and ab are also in $\mathbb{Z}$.

   Alternate wording:

   (h) **The Division Algorithm** For every $a \in \mathbb{Z}$ and $b \in \mathbb{N} - \{0\}$, there exists unique integers q and r such that

2. Outline for a **Direct Proof**

> **Proposition:** If P, then Q.
> **Proof:** (direct) Suppose P (is true).
>
> $\vdots$
>
> Thus, Q (is true). $\square$

3. Prove that for every integer m, if n is even, then $3n^2 - 5mn - 8$ is also even.

4. Let $x, y \in \mathbb{R}^+$. Prove that if $x \leq y$, then $\sqrt{x} \leq \sqrt{y}$.

5. Rigid and Unforgiving Rules

   (a) All parts of all proofs are complete sentences which begin with a word in English and end with a period. No sentence fragments.

   (b) The following symbols never appear: $\forall$, $\exists$, $\Rightarrow$, $\vee$, $\wedge$.

   (c) **All** strings of equalities are aligned vertically, with justification.

6. Let $a, b \in \mathbb{Z}$. Prove that if $a \big| b$, then $a^2 \big| b^2$.

7. Let $x, y \in \mathbb{R}^+$. Prove that $\sqrt{xy} \leq \frac{x+y}{2}$. (Hint at the bottom of the paper.)

Start with what you know about $(x-y)^2$.