INTRODUCTION TO THE DETERMINANT

1. Make a list of statements equivalent to: **The $n \times n$ matrix A is nonsingular.**

2. Make the analogous list for: **The $n \times n$ matrix A is singular.**

3. Cor 4.11 (pg 259) The matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is nonsingular if and only if

The Definition of the Determinant (from 4.3.1 text)

4. **definition:** Let $A = (a_{ij})$ be an $n \times n$ matrix. Then A_{ij} is the $(n-1) \times (n-1)$ matrix obtained by deleting the ith row and jth column from A.

5. **example:** Find B_{23} for $B = \begin{pmatrix} 1 & 2 & 4 \\ 4 & 1 & -1 \\ 2 & -3 & 5 \end{pmatrix}$.

6. **definition:** The *determinant* of the $n \times n$ matrix $A = (a_{ij})$, denoted by $det(A)$, is defined as follows.

   - For $A = (a_{11})$, $det(A) = a_{11}$
   - For $n \geq 2$, $det(A)$ is the sum of n terms of the form $\pm a_{1j}\, det(A_{1j})$, with the plus and minus signs alternating starting with a plus. Specifically,

$$det(A) = a_{11}\, det(A_{11}) - a_{12}\, det(A_{12}) + a_{13}\, det(A_{13}) - \cdots + (-1)^{n-1} a_{1n}\, det(A_{1n})$$

7. Find $det(B)$.