Where would this definition come from?

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \text{nonsingular.}$$

So we want $\det(A) \neq 0$. (!!)

We know $\operatorname{rref}(A) = I_n$. Start this process.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \xrightarrow[a_{11}r_3 \mapsto r_3]{a_{11}r_2 \mapsto r_2} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{11}a_{21} & a_{11}a_{22} & a_{11}a_{23} \\ a_{11}a_{31} & a_{11}a_{32} & a_{11}a_{33} \end{bmatrix}$$

$$\xrightarrow[r_3 - a_{31}r_1 \mapsto r_3]{r_2 - a_{21}r_1 \mapsto r_2} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{11}a_{22} - a_{21}a_{12} & a_{11}a_{23} - a_{21}a_{13} \\ 0 & a_{11}a_{32} - a_{31}a_{12} & a_{11}a_{33} - a_{31}a_{13} \end{bmatrix}$$

$$\xrightarrow[r_3 - a_{31}r_1 \mapsto r_3]{r_2 - a_{21}r_1 \mapsto r_2} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & \boxed{a_{11}a_{22} - a_{21}a_{12}} & a_{11}a_{23} - a_{21}a_{13} \\ 0 & \boxed{a_{11}a_{32} - a_{31}a_{12}} & a_{11}a_{33} - a_{31}a_{13} \end{bmatrix}$$

Observe that one of these two must be nonzero because... A is nonsingular.

We can assume entry (2,2) is not zero.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{11}a_{22} - a_{21}a_{12} & a_{11}a_{23} - a_{21}a_{13} \\ 0 & a_{11}a_{32} - a_{31}a_{12} & a_{11}a_{33} - a_{31}a_{13} \end{bmatrix}$$

$$\Big\downarrow\ (a_{11}a_{22} - a_{21}a_{12})r_3 - (a_{11}a_{32} - a_{31}a_{12})r_2 \mapsto r_3$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{11}a_{22} - a_{21}a_{12} & a_{11}a_{23} - a_{21}a_{13} \\ 0 & 0 & a_{11}\Delta \end{bmatrix}$$

where

$$\Delta = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

OR

$$\Delta = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{21} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{32} \\ a_{22} & a_{31} \end{vmatrix}$$