Name: Solutions ______ / 10

There are 10 points possible on this quiz. You may use technology but you must demonstrate what you are using technology for.

Questions below concern bases of $\mathbb{R}^2$

$$\mathscr{E}_2 = \left\langle \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix} \right\rangle, \qquad B = \left\langle \begin{pmatrix}0\\-1\end{pmatrix}, \begin{pmatrix}2\\1\end{pmatrix} \right\rangle, \qquad D = \left\langle \begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}-1\\1\end{pmatrix} \right\rangle.$$

1. (2 points) Find **directly** the representation of the vector $\vec{v}$ with respect to $\mathscr{E}_2$ assuming $Rep_B(\vec{v}) = \begin{pmatrix}2\\3\end{pmatrix}_B$.

$$\begin{pmatrix}2\\3\end{pmatrix}_B = 2\begin{pmatrix}0\\-1\end{pmatrix} + 3\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}0\\-2\end{pmatrix} + \begin{pmatrix}6\\3\end{pmatrix} = \begin{pmatrix}6\\1\end{pmatrix}$$

2. (2 points) Find the matrix $A_1 = Rep_{B,\mathscr{E}_2}(id)$.

$$A_1 = \begin{bmatrix}0 & 2\\-1 & 1\end{bmatrix}$$

$$A_1 (\vec{v})_B = \begin{bmatrix}0 & 2\\-1 & 1\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix}_B = \begin{pmatrix}6\\1\end{pmatrix} \checkmark$$

3. (1 point) Use matrix A_1 to find $Rep_{\mathscr{E}_2}(\vec{v})$.

4. (5 points) Suppose that, with respect to $\mathcal{E}_2$ for both domain and codomain, the transformation $h : \mathbb{R}^2 \to \mathbb{R}^2$ is represented by the matrix $H = \begin{pmatrix} 6 & -8 \\ 2 & -8 \end{pmatrix}$. Use change of basis matrices to represent h with respect to the bases

$$B = \left\langle \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\rangle \quad \text{and} \quad D = \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\rangle.$$

where B is the basis for the domain and D is the basis for the range.

$M_1 = \begin{bmatrix} 0 & 2 \\ -1 & 1 \end{bmatrix}$ changes B to $\mathcal{E}_2$.

Need M_3 changes $\mathcal{E}_2$ to D.

$$\left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & -1/2 & 1/2 \end{array}\right], \quad M_3 = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

$D \leftarrow \mathcal{E}_2$

$$\text{Rep}_{B,D}(h) = M_3 \cdot H \cdot M_1 = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 6 & -8 \\ 2 & -8 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ 0 & -4 \end{bmatrix}$$