Name: Solutions ________ / 10

There are 10 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. (2 points) Formally state the definition of a **nonsingular matrix**.

A square matrix is nonsingular if it is the coefficient matrix of a homogeneous system of equations with a unique solution

2. (4 points) Determine if the matrix below is singular or nonsingular. Explain your answer.

$$\begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & 2 & 3 \end{bmatrix} \xrightarrow{r_1+2r_2 \mapsto r_1} \begin{bmatrix} 0 & 3 & 2 \\ -1 & 1 & 1 \\ 0 & 2 & 3 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_2} \begin{bmatrix} -1 & 1 & 1 \\ 0 & 3 & 2 \\ 0 & 2 & 3 \end{bmatrix}$$

$$\xrightarrow{-\frac{2}{3} r_2 + r_3 \mapsto r_3} \begin{bmatrix} -1 & 1 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & \frac{2}{3} \end{bmatrix}.$$

So if this was a coeff matrix of homogeneous system $x=y=z=0$, a unique solution.

Thus, the matrix is nonsingular.

3. (4 points) Below you are given a system of linear equations, the matrix form of the system (matrix A), and an echelon from of matrix A (matrix B). Find a solution to the system of linear equations. Express the solution set using vectors. Identify a particular solution and identify the solution set of the homogeneous system.

$$\begin{cases} w+3x+\ y \qquad\ \ =5 \\ w+3x+2y+4z=4 \\ 2w+6x+4y+8z=8 \end{cases} \qquad A=\begin{bmatrix} 1 & 3 & 1 & 0 & 5 \\ 1 & 3 & 2 & 4 & 4 \\ 2 & 6 & 4 & 8 & 8 \end{bmatrix} \qquad B=\begin{array}{c} \begin{array}{ccccc} w & x & y & z & \ \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & -4 & 6 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$w+3x-4z=6$$
$$y+4z=-1$$

$$y=-4z-1$$
$$w=-3x+4z+6$$

$$\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3x+4z+6 \\ x \\ -4z-1 \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} 6 \\ 0 \\ -1 \\ 0 \end{pmatrix}}_{\text{particular}} + \underbrace{\begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} x + \begin{pmatrix} 4 \\ 0 \\ -4 \\ 1 \end{pmatrix} z}_{\text{homogeneous}}$$

Proper answer:

Solution set $\left\{ \begin{pmatrix} 6 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} x + \begin{pmatrix} 4 \\ 0 \\ -1 \\ 1 \end{pmatrix} z : x, z \in \mathbb{R} \right\}$