**Name:** ______________________________ ________ / 10

There are 10 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. (5 points) Find the reduced row echelon form of the matrix below. Show your steps.

$$\begin{bmatrix} 0 & 0 & 1 & -4 \\ 0 & 2 & -3 & 7 \\ 2 & 8 & 4 & 2 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{bmatrix} 2 & 8 & 4 & 2 \\ 0 & 2 & -3 & 7 \\ 0 & 0 & 1 & -4 \end{bmatrix} \xrightarrow{\frac{1}{2}r_1 \to r_1} \begin{bmatrix} 1 & 4 & 2 & 1 \\ 0 & 2 & -3 & 7 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

$$\xrightarrow[r_1 - 2r_3 \to r_1]{r_2 + 3r_3 \to r_2} \begin{bmatrix} 1 & 4 & 0 & 9 \\ 0 & 2 & 0 & -5 \\ 0 & 0 & 1 & -4 \end{bmatrix} \xrightarrow{r_1 - 2r_2 \to r_1} \begin{bmatrix} 1 & 0 & 0 & 19 \\ 0 & 2 & 0 & -5 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}r_2 \to r_2} \begin{bmatrix} 1 & 0 & 0 & 19 \\ 0 & 1 & 0 & -5/2 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

**1.1 Definition** A *vector space* (over $\mathbb{R}$) consists of a set $V$ along with two operations '+' and '·' subject to the conditions that for all vectors $\vec{v}, \vec{w}, \vec{u} \in V$ and all *scalars* $r, s \in \mathbb{R}$:

(1) the set $V$ is closed under vector addition, that is, $\vec{v} + \vec{w} \in V$
(2) vector addition is commutative, $\vec{v} + \vec{w} = \vec{w} + \vec{v}$
(3) vector addition is associative, $(\vec{v} + \vec{w}) + \vec{u} = \vec{v} + (\vec{w} + \vec{u})$
(4) there is a *zero vector* $\vec{0} \in V$ such that $\vec{v} + \vec{0} = \vec{v}$ for all $\vec{v} \in V$
(5) each $\vec{v} \in V$ has an *additive inverse* $\vec{w} \in V$ such that $\vec{w} + \vec{v} = \vec{0}$
(6) the set $V$ is closed under scalar multiplication, that is, $r \cdot \vec{v} \in V$
(7) scalar multiplication distributes over scalar addition, $(r+s) \cdot \vec{v} = r \cdot \vec{v} + s \cdot \vec{v}$
(8) scalar multiplication distributes over vector addition, $r \cdot (\vec{v} + \vec{w}) = r \cdot \vec{v} + r \cdot \vec{w}$
(9) ordinary multiplication of scalars associates with scalar multiplication, $(rs) \cdot \vec{v} = r \cdot (s \cdot \vec{v})$
(10) multiplication by the scalar 1 is the identity operation, $1 \cdot \vec{v} = \vec{v}$.

2. (5 points) Let $k$ be a fixed real number and $V = \{f(x) : \mathbb{R} \to \mathbb{R} : f(x) = Ce^{kx}, C \in \mathbb{Z}^+ \cup \{0\}\}$ under the usual function addition and scalar multiplication. (So, $V$ is the set of all real-valued functions of the form $f(x) = Ce^{kx}$ where $C \in \{0, 1, 2, 3, 4, \ldots\}$.)

(a) Show that $V$ satisfies (1) and (4) in the definition of a vector space.

(1) Pick $c_1e^{kx}, c_2e^{kx}$. Then $c_1e^{kx} + c_2e^{kx} = (c_1+c_2)e^{kx}$.
Since $c_1, c_2 \in \mathbb{Z}^+ \cup \{0\}$, then $c_1 + c_2 \in \mathbb{Z}^+ \cup \{0\}$. So $(c_1+c_2)e^{kx} \in V$.

(2) $0 \in \mathbb{Z}^+ \cup \{0\}$. So $0 \cdot e^{kx}$ is in $V$.

Now, $0e^{kx} = 0$ which is the additive identity in $V$.

(b) Show that $V$ is not a vector space.

$10e^{kx} \in V$ and $-1 \in \mathbb{R}$ but $-1 \cdot 10e^{kx} = -10e^{kx} \notin V$
since $-10 \notin \mathbb{Z}^+ \cup \{0\}$.