

Name: _____ / 10

There are 10 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. (5 points) Find the reduced row echelon form of the matrix below. Show your steps.

$$\begin{bmatrix} 0 & 0 & 1 & -4 \\ 0 & 2 & -3 & 7 \\ 2 & 8 & 4 & 2 \end{bmatrix}$$

1.1 Definition A *vector space* (over \mathbb{R}) consists of a set V along with two operations '+' and '·' subject to the conditions that for all vectors $\vec{v}, \vec{w}, \vec{u} \in V$ and all *scalars* $r, s \in \mathbb{R}$:

- (1) the set V is closed under vector addition, that is, $\vec{v} + \vec{w} \in V$
- (2) vector addition is commutative, $\vec{v} + \vec{w} = \vec{w} + \vec{v}$
- (3) vector addition is associative, $(\vec{v} + \vec{w}) + \vec{u} = \vec{v} + (\vec{w} + \vec{u})$
- (4) there is a *zero vector* $\vec{0} \in V$ such that $\vec{v} + \vec{0} = \vec{v}$ for all $\vec{v} \in V$
- (5) each $\vec{v} \in V$ has an *additive inverse* $\vec{w} \in V$ such that $\vec{w} + \vec{v} = \vec{0}$
- (6) the set V is closed under scalar multiplication, that is, $r \cdot \vec{v} \in V$
- (7) scalar multiplication distributes over scalar addition, $(r + s) \cdot \vec{v} = r \cdot \vec{v} + s \cdot \vec{v}$
- (8) scalar multiplication distributes over vector addition, $r \cdot (\vec{v} + \vec{w}) = r \cdot \vec{v} + r \cdot \vec{w}$
- (9) ordinary multiplication of scalars associates with scalar multiplication, $(rs) \cdot \vec{v} = r \cdot (s \cdot \vec{v})$
- (10) multiplication by the scalar 1 is the identity operation, $1 \cdot \vec{v} = \vec{v}$.

2. (5 points) Let k be a fixed real number and $V = \{f(x) : \mathbb{R} \rightarrow \mathbb{R} : f(x) = Ce^{kx}, C \in \mathbb{Z}^+ \cup \{0\}\}$ under the usual function addition and scalar multiplication. (So, V is the set of all real-valued functions of the form $f(x) = Ce^{kx}$ where $C \in \{0, 1, 2, 3, 4, \dots\}$.)

(a) Show that V satisfies (1) and (4) in the definition of a vector space.

(b) Show that V is not a vector space.