Name: Solutions ________ / 10

There are 10 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. (5 points) Do the vectors $S = \left\{ \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ span the vector space $M_{2\times 2}$, the vector space of all 2 by 2 matrices? Justify your answer.

Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be an arbitrary element of $M_{2\times 2}$.

Can we find constants c_1, c_2, c_3, c_4 so that

$$c_1\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + c_2\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} + c_3\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + c_4\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ ?}$$

Find c_i's so that

$$\begin{aligned} c_1 + c_2 + c_3 &= a \\ c_1 + c_2 + c_3 &= b \\ c_2 + c_3 &= c \\ c_3 + c_4 &= d \end{aligned} \quad \text{or} \quad \left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & a \\ 1 & 1 & 1 & 0 & b \\ 0 & 1 & 1 & 0 & c \\ 0 & 0 & 1 & 1 & d \end{array}\right) \xrightarrow{\text{rref}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & * \\ 0 & 1 & 0 & -1 & * \\ 0 & 0 & 1 & 1 & * \\ 0 & 0 & 0 & 0 & * \end{array}\right)$$

The row of zeros suggests that there are choices for a, b, c, d for which no c_i's are possible. Pick $(a,b,c,d) = (1,2,3,4)$.

Now

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 & 4 \end{array}\right) \xrightarrow{\text{rref}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right) = B$$

Since the last row of B corresponds to the equation $0 = 1$, the system has no solution. Thus, the original has no solution. Thus, the set of vectors do <u>NOT</u> span $M_{2\times 2}$.

2. (5 points) Parametrize the subspace $W = \{\ a+bx+cx^2+dx^3\ :\ a=b,\ c=2d \text{ and } a,b,c,d \in \mathbb{R}\}$. Then express the subspace as a span.

$$W = \{ a+bx+cx^2+dx^3 : a=b;\ c=2d;\ a,b,c,d \in \mathbb{R} \}$$

$$= \{ a+ax+2dx^2+dx^3 : a,d \in \mathbb{R} \}$$

$$= \{ a(1+x)+d(2x^2+x^3) : a,d \in \mathbb{R} \}$$ ← parametrize

$$= \text{span}(\{ 1+x, 2x^2+x^3 \})$$ ← express as a span