Name: Solutions ______ / 10

There are 10 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. (5 points) Do the vectors $\{1+x, x-2x^2, x^2\}$ span the vector space $\mathscr{P}_2$, the vector space of all polynomials of degree 2 or less? Justify your answer.

Let $a_0 + a_1x + a_2x^2$ be an arbitrary element of P_2.

We need to find $c_1, c_2, c_3 \in \mathbb{R}$ so that

$$c_1(1+x) + c_2(x-2x^2) + c_3(x^2) = a_0 + a_1x + a_2x^2.$$

Or, equivalently:

$$c_1 + (c_1+c_2)x + (-2c_2+c_3)x^2 = a_0 + a_1x + a_2x^2.$$

Or, equivalently,

$$\begin{aligned} c_1 &= a_0 \\ c_1 + c_2 &= a_1 \\ -2c_2 + c_3 &= a_2 \end{aligned}$$

Strategy 1: Explicitly state a general solution

$\boxed{c_1 = a_0}$, $c_2 = a_1 - c_1 = a_1 - a_0$, $c_3 = a_2 + 2c_2 = a_2 + 2(a_1 - a_0)$

$\boxed{\text{Answer Yes}}$

Strategy 2: Demonstrate the existence of a general solution

Observe $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \xrightarrow{r_2 - r_1 \mapsto r_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \xrightarrow{r_3 + 2r_2 \mapsto r_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Since $\left[\begin{array}{ccc|c} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \end{array}\right]$ will always have a solution, so will the original system. $\boxed{\text{Answer: Yes}}$

2. (5 points) Parametrize the subspace $W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a+b+c=0 \right\}$. Then express the subspace as a span.

$a=-b-c$. So:

$$W=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a+b+c=0\right\}=\left\{\begin{pmatrix} -b-c & b \\ c & d \end{pmatrix} : b,c,d\in\mathbb{R}\right\}$$

$$=\left\{b\begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}+c\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}+d\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} : b,c,d\in\mathbb{R}\right\}$$

$$=\operatorname{span}\left(\left\{\begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix},\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix},\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right\}\right)$$

1 point Extra Credit Explain why, given your work in problem 2 above, you did not need to be told that W is a subspace.

We described W as the set of all linear combinations of a set of vectors, which is always a subspace. (ie span(S) is always a vector space.)