Name: Solutions ______ / 10

There are 10 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. (1 point) Complete the following statement:

   A subset $S = \{\vec{s}_1, \vec{s}_2, \cdots, \vec{s}_n\}$ of a vector space is linearly independent if and only if $c_1\vec{s}_1 + c_2\vec{s}_2 + \cdots c_n\vec{s}_n = \vec{0}$

   has only 1 solution.

2. (3 points) Show that the set $S = \{f(x) = x^2, g(x) = e^x\}$ is linearly independent in the vector space of all functions from $\mathbb{R}$ to $\mathbb{R}$.

Consider $c_1 x^2 + c_2 e^x = 0$.

Pick $x=0$ and $x=1$ to obtain $c_2 = 0$ and $c_1 + ec_2 = 0$.

So $c_2 = 0$ and $c_1 = 0$. Thus, S is linearly independent.

3. (1 point) State the definition of a basis. of V

B is a sequence so that B is linearly independent and spans V.

4. (4 points) It is a fact that $B = \langle \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix} \rangle$ is a basis for $\mathscr{M}_{2\times 2}$ the vector space of all 2 by 2 matrices.

(a) Confirm that B spans $\mathscr{M}_{2\times 2}$.

We need c_i's so that $c_1\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + c_2\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + c_3\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + c_4\begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{aligned} c_1 + c_2 \quad &= a \\ c_2 \quad &= b \\ c_3 + c_4 &= c \\ -c_4 &= d \end{aligned}$$

So $c_1 = a-b,\ c_2 = b,\ c_3 = c+d,\ c_4 = -d$

So B spans $M_{2\times 2}$.

(b) Represent $\vec{v} = \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix}$ with respect to the basis B.

$c_1 = 1-2 = -1$

$c_2 = 2$

$c_3 = -3+4 = 1$

$c_4 = -4$

$\mathrm{rep}_B(\vec{v}) = \begin{pmatrix} -1 \\ 2 \\ 1 \\ -4 \end{pmatrix}$