Name: Solutions ______ / 10

There are 10 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. (6 points) Let $V = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{R} \right\}$ and $G : V \to \mathscr{P}_2$ be defined by $G\left(\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}\right) = a + bx + (b + d)x^2$.

   (a) Show that G is an onto function.

   Pick an arbitrary $c_1 + c_2 x + c_3 x^2$ in $\mathscr{P}_2$.

   Then $\begin{pmatrix} c_1 & c_2 \\ 0 & c_3 - c_2 \end{pmatrix}$ is in V and $G\left(\begin{pmatrix} c_1 & c_2 \\ 0 & c_3 - c_2 \end{pmatrix}\right) = c_1 + c_2 x + (c_2 + c_3 - c_2)x^2$

   $= c_1 + c_2 x + c_3 x^2$

   So G is onto.

   (b) Show that G respects vector addition.

   Pick $\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$ and $\begin{pmatrix} a' & b' \\ 0 & d' \end{pmatrix}$ to be arbitrary elements of V.

   Then $G\left(\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} + \begin{pmatrix} a' & b' \\ 0 & d' \end{pmatrix}\right) = G\left(\begin{pmatrix} a+a' & b+b' \\ 0 & d+d' \end{pmatrix}\right) = (a+a') + (b+b')x + (b+b'+d+d')x^2$

   $= \left(a + bx + (b+d)x^2\right) + \left(a' + b'x + (b'+d')x^2\right)$

   $= G\left(\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}\right) + G\left(\begin{pmatrix} a' & b' \\ 0 & d' \end{pmatrix}\right)$

2. (4 points) Explain why each of the functions below fails to be an isomorphism.

(a) $f : \mathscr{M}_{2\times 2} \to \mathbb{R}$ defined by $f\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = ad - bc$.

f is not 1-1.

Observe that $f\left(\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}\right) = 0$ and $f\left(\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\right) = 0$.

(b) $f : \mathbb{R}^2 \to \mathbb{R}^2$ by $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2y+1 \\ -x \end{bmatrix}$.

f does not map the zero vector to the zero vector.

Specifically, $f\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.