

Name: _____ / 10

There are 10 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. (6 points) Let $V = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{R} \right\}$ and $G: V \rightarrow \mathcal{P}_2$ be defined by $G\left(\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}\right) = a + bx + (b+d)x^2$.

(a) Show that G is an onto function.

(b) Show that G respects vector addition.

2. (4 points) Explain why each of the functions below fails to be an isomorphism.

(a) $f : \mathcal{M}_{2 \times 2} \rightarrow \mathbb{R}$ defined by $f \left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} \right) = ad - bc$.

(b) $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by $f \left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} 2y + 1 \\ -x \end{bmatrix}$.