Name: Solutions _____ / 10

There are 10 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **This is a short-answer quiz.**

1. (2 points)

   (a) What is the null space of the differentiation transformation $d/dx : \mathscr{P}_n \to \mathscr{P}_n$?

   $S = \{k : k \in \mathbb{R}\}$ = the set of constant polynomials

   (b) What is the rank of the differentiation transformation $d/dx : \mathscr{P}_n \to \mathscr{P}_n$?

   $\dim(P_n) = n$

   $\dim(S) = 1$

   So $\dim\left(R\left(\frac{d}{dx}\right)\right)$ = rank of $\frac{d}{dx} = n-1$

2. (4 points) Multiply the matrix $M = \begin{pmatrix} 1 & -1 \\ 2 & 0 \\ 0 & -3 \end{pmatrix}$ by each vector below or state that the operation is not defined.

   (a) $\vec{v} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$ not defined

   (b) $\vec{v} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

   $$\begin{pmatrix} 1 & -1 \\ 2 & 0 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 2+3 \\ 4 \\ 0+9 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 9 \end{pmatrix}$$

3. (4 points) Consider the linear map $h : V \to W$ represented with respect to some bases B, D by the matrix $M = \begin{pmatrix} 1 & 1 & 0 & -2 \\ 2 & 3 & 1 & -1 \\ 1 & 2 & 1 & 1 \end{pmatrix}$. Observe that the reduced echelon form of M is $\begin{pmatrix} 1 & 0 & -1 & -5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

   (a) What is the dimension of the domain of h?

   4

   (b) What is the dimension of the codomain of h?

   3

   (c) What is the dimension of the range of h?

   $\dim \text{range} = \text{rank } h = \text{rank } M = \text{rank}(\text{rref}(M)) = 2$

   (d) What is the dimension of the nullity of h?

   $4 - 2 = 2$