Midterm II Review

Logistics:
The midterm will be one hour. You may bring in a single sheet of hand written notes. You should bring some form of technology that will allow you to: put a matrix into reduced row echelon form and to multiply matrices.
There will be one problem for which you must demonstrate that you know how to multiply matrices by hand.

Section 3.1.1: Definition of Isomorphisms

Terminology: isomorphism, image

Lemmas/Theorems:
Lemma 1.10: An isomorphism maps the zero vector of the domain to the zero vector of the codomain.
Lemma 1.11: This give alternate ways to demonstrate a map is linear.

Sample Problems:
Given a map, find the image of elements in the domain (1.13).
Verify that a given map is (or is not) an isomorphism (1.16, 1.17).

Section 3.1.2: Dimension Characterizes Isomorphism

Lemmas/Theorems:
Theorem 2.3: Vector spaces are isomorphic if and only if they have the same dimension.

Sample Problems:
Observe that you can determine if two vector spaces are or are not isomorphic based on their dimension. (2.10)

Section 3.2.1: Definition of Homomorphism

Terminology: homomorphism, zero homomorphism, linear extension of a map

Lemmas/Theorems:
Lemma 1.6: A linear map send the zero vector of the domain to the zero vector of the range.
Theorem 1.9: A homomorphism is determined by the action on a basis.

Sample Problems:
Determine whether or not a given map is linear (1.18, 1.19)
Verify the a particular map is a homomorphism (1.22)

Section 3.2.2: Range Space and Null Space

Terminology: range space, rank of a homomorphism, null space, nullity of a homomorphism, image (again), inverse image

Lemmas/Theorems:
Lemma 2.1: Under a homomorphism, the image of a subspace of the domain is a subspace of the codomain. In particular, the range of the homomorphism is a subspace of the codomain.
Lemma 2.10: For any homomorphism, the inverse image of a subspace of the range is a subspace of the domain. In particular, the inverse image of the zero vector in the range is a subspace (ie null space) of

the domain.
Theorem 2.14: For any linear map, the rank of the map plus the nullity of the map must equal the dimension of the domain.
Corollary 2.17: The rank of a linear map is less than or equal to the dimension of the domain and equality holds if and only if the nullity is zero.
Lemma 2.18: Under a linear map, the image of a set of linearly dependent vectors must be linearly dependent.
Theorem 2.20: This is a "The following are equivalent..." list starting with the statement "$h : V \to W$ is one to one" where $dim(V) = n$.

Sample Problems:
Determine whether or not a vector is in the range or null space of a linear map. (2.21)
Determine the range space or null space of a linear map (and thus the rank and nullity of the map). (2.23, 2.24, 2.25)
Find the inverse image of a vector (2.30)

Section 3.3.1: Representing Linear Maps with Matrices

Terminology: the matrix representation of a linear map with respect to bases for domain and codomain ($Rep_{B,D}(h)$), the matrix-vector product (which we now interpret as matrix multiplication...)

Sample Problems:
How to use the matrix representation to find the image of a vector. (1.13)
How to use the image of basis elements to find the image of a vector (1.17)
Find the matrix representation of a linear map with respect to given bases (1.19, 1.21, 1,27*) .
*Note that because of sections 3.5.1 and 3.5.2, we now have a different way of thinking about 1.27.

Section 3.3.2: Any Matrix Can Represent a Linear Map

Terminology: a nonsingular linear map

Lemmas/Theorems:
Theorem 2.2: Any matrix can be interpreted as a representation of a linear map. The representation is not unique as it depends on the bases.
Quick Reminder: If H is an $m \times n$ matrix that represents a linear map from V to W, then $dim(W) = m$ and $dim(V) = n$.
Theorem 2.4: The rank of a matrix equal the rank of any linear map it represents.
Corollary 2.6: Let h be a linear map represented by matrix H. The map h is onto if and only if rank of H equal the number of rows of H. The map h is one to one if and only if rank of H equal the number of columns of H.

Sample Problems:
Determine if a map is singular or nonsingular from its matrix representation (2.13)
Determine the image of a vector given a matrix representation and corresponding bases. (2.14, 2.15, 2.16)

Section 3.4.1: Sums and Scalar Products of Matrices

Terminology: sum of two matrices, scalar multiple of two matrices

Lemmas/Theorems:

Theorem 1.4: This states the expected relationship between matrix addition/scalar multiplication of matrices to that of linear maps.

Sample Problems:
Be able to perform matrix addition and scalar multiplication of matrices.

Section 3.4.2: Matrix Multiplication

Terminology: matrix multiplication

Lemmas/Theorems:
Theorem 2.7: A composition of linear maps is represented by matrix multiplication of their respective representations.
Theorem 2.12: Matrix multiplication (if defined) is associative and has the expected distributive laws.

Sample Problems:
Be able to perform matrix multiplication by hand. (2.14)
Be able to find the matrix of a composition of linear functions using matrix representations. (2.19)

Section 3.4.3: Mechanics of Matrix Multiplication

Terminology: main diagonal, identity matrix, diagonal matrix, permutation matrix, elementary reduction matrices

Lemmas/Theorems:
Corollary 3.23: Elementary row and column operations can be performed with a sequence of products of elementary reduction matrices

Sample Problems:
Find the elementary reduction matrix such that appropriate multiplication performs a specific row or column operation. (3.27)

Section 3.4.4: Inverses

Terminology: invertible matrix,

Lemmas/Theorems:
Theorem 4.3: A matrix is invertible if and only if it is nonsingular.
Lemma 4.7: A matrix is invertible if and only if it can be written as a product of elementary reduction matrices.

Sample Problems:
Be able to find an inverse of a matrix *using only the reduced row echelon operator* and be able to determine that no inverse exists (4.15).
Be able to use matrix algebra (and inverses) to solve matrix problems. (4.18)

Section 3.5.1 & 3.5.2: Changing Representations of Vectors and Linear Maps

Terminology: change of basis matrix ($Rep_{B,D}(id)$), matrix representation of a linear map with respect to given bases (again) $Rep_{B,D}(id)$

Lemmas/Theorems:

Lemma 1.5: A matrix is a change of basis matrix if and only if it is nonsingular.

Sample Problems:

Find the change of basis matrix ($Rep_{B,D}(id)$) given B and D. (S 3.5.1 #1.9)
Be able to use a change of basis matrix (S 3.5.1 #1.12)
Be able to change the matrix representation of a linear map from one set of bases to another (S 3.5.2 #2.14, 2.17)