Notes from Fri 9 Sept.

I. Recap of One.I.2 thus far

- The number of solutions in the set of all solutions of a system of linear equations will be

- Every system of linear equations will have a solution set with form

- Every system of linear equations will have a solution set with form

$$\{\vec{p} + \vec{h} : \vec{h} \text{ soln. to homogeneous system}\}$$

Example of this Thm in action.

SoLE: $\begin{aligned} x + 2y - z &= 2 \\ 2x - y - 2z + w &= 5 \end{aligned}$

(i) Observe that $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 12/5 \\ -1/5 \\ 0 \\ 0 \end{pmatrix}$ is a (particular) solution (ie a "$\vec{p}$")

(ii) Solve homogeneous system

$$\begin{aligned} x + 2y - z &= 0 \\ 2x - y - 2z + w &= 0 \end{aligned}$$

$$\left[\begin{array}{cccc:c} 1 & 2 & -1 & 0 & 0 \\ 2 & -1 & -2 & 1 & 0 \end{array}\right] \xrightarrow{-2e_1 + e_2 \mapsto e_2} \left[\begin{array}{cccc:c} 1 & 2 & -1 & 0 & 0 \\ 0 & -5 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{aligned} x + 2y - z &= 0 \\ -5y + w &= 0 \end{aligned}$$

So $y = w/5$

$x = -2y + z = \frac{-2}{5}w + z$

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} \frac{-2}{5}w + z \\ w/5 \\ z \\ w \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} z + \begin{pmatrix} -2/5 \\ 1/5 \\ 0 \\ 1 \end{pmatrix} w$$

Solution to the Original SOLE.

The last of One.I.2

definition: A square matrix is nonsingular

if it is the coefficient matrix

of a homogeneous system with a unique solution.

Otherwise, the matrix is singular.

Examples

SoLE

A $\begin{aligned} x+y &= 7 \\ x+2y &= 4 \end{aligned}$

matrix form $\left[\begin{array}{cc|c} 1 & 1 & 7 \\ 1 & 2 & 4 \end{array}\right]$

Coefficient matrix $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

B $\begin{aligned} x+2y-z &= 2 \\ 2x-y-2z+w &= 5 \end{aligned}$

$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2 \\ 2 & -1 & -2 & 1 & 5 \end{array}\right]$ $\begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & -1 & -2 & 1 \end{bmatrix}$

Is $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ nonsingular?

Does the SoLE $\begin{aligned} x+y &= 0 \\ x+2y &= 0 \end{aligned}$ have a unique solution?

Let's make 3×3 matrices that are

① nonsingular

② singular